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Dynamic base-age invariant site index models based on generalized algebraic difference approach for mixed Scots pine (*Pinus sylvestris* L.) and Oriental beech (*Fagus orientalis* Lipsky) stands

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Abstract: Data from stem analysis of 397 dominant trees (198 Scots pine and 199 Oriental beech) were used to evaluate dynamic base-age invariant site index models derived from Bertalanffy–Richards, Hossfeld, and Lundqvist–Korf functions with the generalized algebraic difference approach (GADA) for mixed Scots pine (*Pinus sylvestris* L.) and Oriental beech (*Fagus orientalis* Lipsky) stands. These functions were compared with respect to residuals of these models; specifically, the evaluation criteria were bias ($\bar{E}$), root mean square error (RMSE), absolute mean error (AME), adjusted coefficient of determination (R^2_{adj}), and Akaike information criterion. The best results were obtained with generalized algebraic difference equations derived from the base models of Bertalanffy–Richards for Oriental beech and Hossfeld for Scots pine. These selected models accounted for 95%–96% of the total variance in height–age relationships in dominant trees with bias of 0.049841 and 0.00171, RMSE of 1.55624 and 1.353736, AME of 0.940128 and 0.884034, and AIC of 723.55 and 1250.78 for Scots pine and Oriental beech, respectively. These dynamic base-age invariant site index models for the 2 tree species presented more effective and accurate polymorphic site index curves with multiple asymptotes than earlier site index models for Oriental beech and Scots pine. The important differences of height growth trend between developed base-age invariant site index models in this study and earlier site curves were determined by graphical comparisons for site index predictions. Therefore, the new dynamic base-age invariant site index models developed based on GADA methodology can be recommended for dominant height prediction and forest site quality evaluations in the mixed stands of these 2 species.

Key words: Site index model, generalized algebraic difference approach, Scots pine, Oriental beech, mixed stands

1. Introduction

Oriental beech (*Fagus orientalis* Lipsky) and Scots pine (*Pinus sylvestris* L.) are 2 common species in the northwest of Turkey. Oriental beech grows naturally in the Black Sea, Marmara, Aegean, and East Mediterranean regions, as well as in many other regions of the world (Davis, 1982). Scots pine (*Pinus sylvestris* L.) is the most widely distributed conifer in the world, with a natural range that stretches from beyond the Arctic Circle in Scandinavia to southern Spain and from western Scotland to the Okhotsk Sea in eastern Siberia. *Pinus sylvestris* var. *hamata* Steven is native to the Balkan Peninsula, the northwest of Turkey, and the southwest of Transcaucasia (Davis, 1982). Scots pine prefers full sun, moist to dry conditions that are well drained, and an acidic sandy soil, although it also adapts to other kinds of soil. Scots pine often forms forests of pure or mixed type, in the higher mountainous areas of North Anatolia; however, it also extends to inner and southern regions in the form of small patches. Scots pine forests in Turkey cover an area of 738,000 ha (General Directorate of Forestry, 2008). Oriental beech (*Fagus orientalis* Lipsky) is a shade-tolerant species, and optimum growth conditions of beech forests are found on the north-facing slopes of the North Anatolian orogenic belt in the Black Sea Region and the Istranca Mountains in Thrace. Oriental beech forests in Turkey cover 1,810,079 ha, composing nearly 8.5% of the country's total forest area (General Directorate of Forestry, 2008). Mixed Scots pine and Oriental beech stands with different forest structures and biodiversity are widespread in the northwest of Turkey. The management of mixed stands of these species is of increasing importance to foresters in Turkey, and a crucial factor is knowledge of site quality estimation for the sound management of these stands.

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Accurate predictions of forest site quality are critical for effective forest management and are useful for stratifying forest areas into productive classes, usually referred to as site classes, by marking them with symbols such as I, II, III, IV, and V (Hamilton and Christie, 1971; Clutter et al., 1983). Estimations of site quality are also an important input variable for forest growth and yield models (Husch et al., 1972). To evaluate forest site quality, the site index, defined as the average height of dominant or co-dominant trees at a specified index age, has been commonly used as a measure of site quality in both pure and mixed stands (Carmean, 1972). A fundamental characteristic of dominant and co-dominant height is its independence from stand density and it not being affected by thinning in silvicultural applications (Clutter et al., 1983; Monserud, 1984).

Even though many methods have been developed for estimating site index and constructing site index curves, Clutter et al. (1983) described 3 general methods having special importance in forestry literature: 1) the guide curve method, 2) the parameter prediction method, and 3) the difference equation method (Palahi et al., 2004). In developing site index prediction systems, some desirable characteristics of site index models are polymorphism, multiple horizontal asymptote, one inflection point, and base-age invariance (Elfving and Kiviste, 1997; Bailey and Cieszewski, 2000). The growth models for site index predictions were obtained by static or dynamic model structures (Diéguez-Aranda et al., 2006). Bailey and Clutter (1974) stated that the general model form of static site index models has Y = (t, S), where Y is the height at age t and S is a fixed-base-age site index. In particular, it is important that a specified base-age must be primarily selected in fitting static equations (e.g., Stage, 1963; Curtis et al., 1974, Monserud, 1984, Bruchwald, 1988), and so base-age variance predictions of these models will highly depend on the preselecting of a fixed base-age in the previous modeling stage (Cieszewski, 2001). However, the dynamic site index models are base-age invariant with a variable base-age, and these models are independent from the selection of base-age specified in the stage of fitting models (Bailey and Cieszewski, 2000; Cieszewski, 2001). Thus, these dynamic models present more advantages for achieving desirable characteristics of growth models than static forms (Diéguez-Aranda et al., 2006; Cieszewski et al., 2007). Cieszewski and Bailey (2000) and Cieszewski (2001, 2003) described the dynamic site index model as having a general form of $Y = f(t, t_0, Y_0)$, where Y is the function's value at t and Y_0 is the reference variable defined as the value of the function at age t_0.

Bailey and Clutter (1974) applied a technique formalizing the base-age invariance feature for dynamic site index models that is called the algebraic difference approach (ADA) in forestry literature. They emphasized that the predictions of a site index model are not affected by the selection of base-age within a single equation. This base-age invariance feature has given a specialized dynamic property to site index models in order to offer some alternative solutions for prediction projections with different base-ages (Cieszewski, 2003). The dynamic site index model derived by the ADA is independent of initial selected base-age, but the ADA can produce a polymorphic site index model with single asymptotes for all productivity sites (Cieszewski, 2001). Efficient and accurate site index predictions have been obtained by the polymorphic site index model with multiple asymptotes. Thus, the need for a polymorphic site index model with multiple asymptotes is becoming more apparent in the development of more advanced polymorphic site index models (Cieszewski, 2003). Cieszewski and Bailey (2000) further prolonged the ADA by introducing the generalized algebraic difference approach (GADA), a generalization of the ADA. The GADA can produce base-age invariant models with multiple asymptotes, and these site index models are capable of describing a polymorphic family of growth curves (Wang et al., 2008). The principal advance of the GADA in site index prediction is that base-height growth equations can be derived to support various theories about the site index model, e.g., asymptotes, polymorphism, and base-age invariance. The dynamic site index models derived by GADA have shown better and more successful properties than the fixed base-age site index models in many studies (e.g., Cieszewski, 2001, 2002, 2003; Cieszewski and Nigh, 2002; Trincado et al., 2003; Diéguez-Aranda et al., 2005, 2006; Nord-Larsen, 2006; Adame et al., 2006; Bravo-Oviedo et al., 2007; Cieszewski et al., 2007; Cieszewski and Strub, 2008; Kitikidou et al., 2012). Thus, GADA-based dynamic site index models have been used commonly for modeling dominant height–age growth and developing site index curves in forestry literature (Cieszewski et al., 2007).

In pure Scots pine and Oriental beech stands, some stand yield models, including site index predictions systems, have been developed and are usually used in Turkey, such as in the studies of Alemdağ (1967), Erdemir (1974), and Ercanlı et al. (2007) for Scots pine and Kalıpsız (1962), Carus (1998), and Atıcı (1998) for Oriental beech. Despite the importance of mixed stands in terms of spatial distributions and ecological, economical, and biological values, only a few studies concerning the assessment of site productivity for mixed stands have been carried out (e.g. Ercanlı, 2010; Yavuz et al., 2011). Thus, the objective of the present study is to develop a dynamic site index system based on the GADA for mixed Oriental beech (*Fagus orientalis* Lipsky) and Scots pine (*Pinus sylvestris* L.) stands in northwestern Turkey.

2. Materials and methods

The data for this study are 166 regional-level sample plots obtained by Kahriman (2011) from mixed Oriental beech (*Fagus orientalis* Lipsky) and Scots pine (*Pinus sylvestris* L.) stands in the northwest of Turkey. The mixed stands studied covered the Almus, Niksar, Erbaa, Bafra, Vezirköprü, Amasya, Kargı, Boyabat, Araç, Kure, Samatlar, Ilgaz, Dirgine, Karabük, and Mengen forest district areas (Figure 1). These sample plots were randomly selected to represent the range of site qualities and age variability throughout these mixed stands. The method of selection for the sample plot was stratified randomly to guarantee that various conditions of productivity and stand structure occurred in these mixed stands. The stratified random sampling procedure were carried out by considering forest management plans that provided information concerning site productivity and stand structures for the studied beech/pine mixed stands.

The altitudes of the studied area varied from 750 to 1750 m and the slope ranged between 5% and 60%. The areas studied were characterized geomorphologically as being high mountainous lands with moderate and steep slopes. The mean annual temperature ranges between –5.8 and 14.6 °C. The climatic regime is a typical Black Sea climate, characterized by a mild winter and a cool summer. The mean annual rainfall varies between 1000 and 1250 mm with relatively homogeneous precipitation.

These sampled mixed stands were naturally regenerated and uniformly stocked stands (60%–90% tree layer cover), without any evidence of historical damage such as fire or storms. The beech/pine mixed stands were selected to have uniform stratification of these species where both of the 2 species have been in the upper stratum, such that there are site trees of both species in the plot. The plot sizes ranged from 0.06 to 0.12 ha, depending on stand density in order to achieve a minimum of 30–40 trees per species in sample plots. Diameter at breast height (1.3 m) and height measurements were collected for all live trees in the sample plots. In this study, 2 dominant and co-dominant trees (1 beech and 1 pine tree) of the 100 trees of greatest height per hectare were sampled for stem analysis in the sample plots. These sample trees had to be dominant or co-dominant free-growing trees with no obvious evidence of growth abnormalities or damages. These trees sampled for stem analysis were also felled, and cross-sectional cuts were made at the first 0.3 m and second 1.3 m of the stem and every 2 m throughout the tree stem. The number of rings was counted at each cross-sectioned point, and then

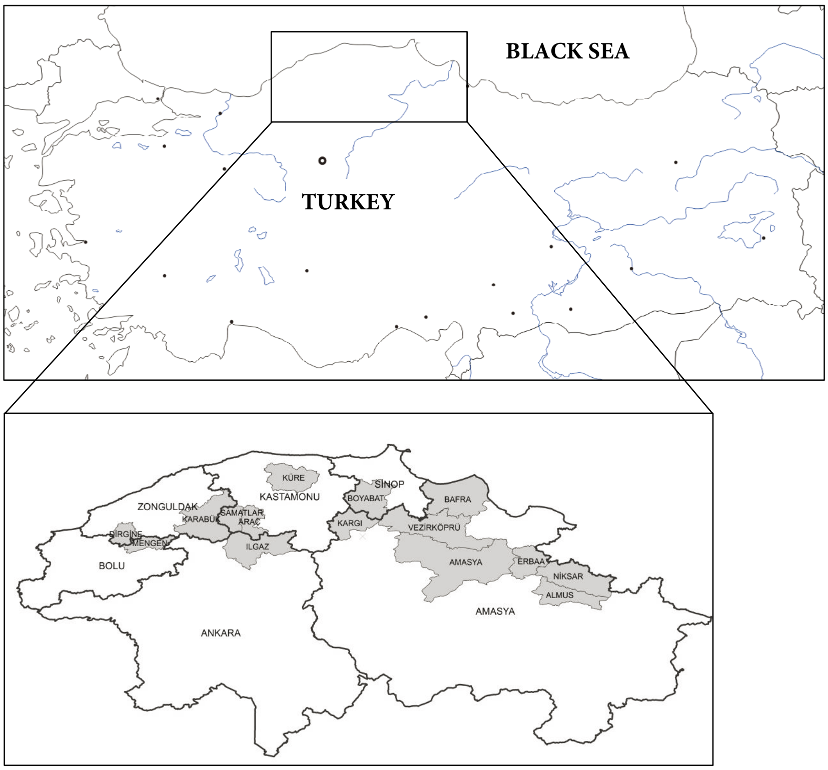


Figure 1. Maps showing the locations of the studied mixed Scots pine and Oriental beech stands.

these values were converted to stump age, which can be considered equal to tree age. As cross-section lengths do not coincide with periodic height growth, it was necessary to adjust height/age data from stem analysis. Fabbio et al. (1994) presented an iterative screening and structure analysis (ISSA) method that used the second differences of ring counts to get smoother height/age curves. The heights for each age of sampled trees were calculated by the ISSA method based on height/age data at cross-sectional cuts. The final data set for modeling and comparing the site index models included a total of 397 stem analyses (198 Scots pine and 199 Oriental beech trees) in this study. A total of 2319 height/age pairs (1217 for Scots pine and 1102 for Oriental beech) were used to fit the dynamic site index equations. The mean, standard deviation, minimum, maximum, and 95% confidence interval for data used are presented in Table 1. The profile plots of the stem analysis for Scots pine and Oriental beech are presented in Figure 2.

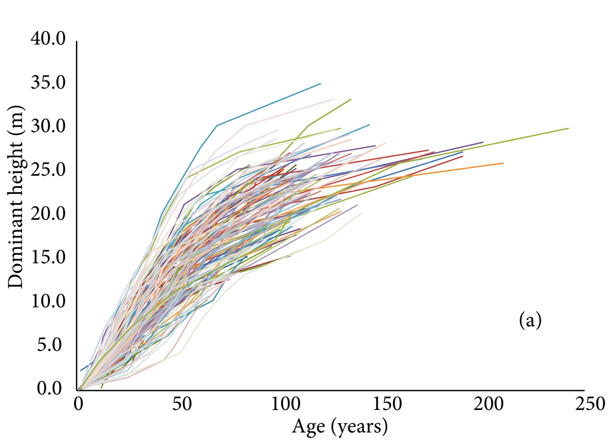
2.1. Functions Selected

A number of statistical growth functions have been used to model the height–age relationship in forestry literature (Gadow and Hui, 1999; Diéguez-Aranda et al., 2006). In particular, the Bertalanffy–Richards (Richards 1959), Hossfeld (Hossfeld, 1882) and Lundqvist–Korf (Lundqvist, 1957) growth models have been the most popular and suitable for site index prediction systems (Pienaar and Turnbull, 1973; Hui and Gadow, 1993; Elfving and Kiviste, 1997). When assessing growth models for site index predictions, the desirable attributes of these models are: 1) polymorphism, 2) base-age invariance, 3) sigmoid growth pattern with an inflexion point 4) passing a zero point, 5) multiple asymptotes with function of site index (increasing with increasing site index), 6) parallel asymptotes at older ages, and 7) a nondecreasing curve over time (Corral-Rivas et al., 2004; Diéguez-Aranda et al., 2006; Martín-Benito et al., 2008). In developing accurate and efficient site quality systems, these attributes of site index predictions have been accomplished and confirmed by using a GADA-based dynamic site index model derived from the base statistical growth functions (Cieszewski et al., 2006).

The GADA methodology presents the dynamic site index models as having 1 or 2 parameters considered to be site-specific in association with site quality (Diéguez-Aranda et al., 2006). In particular, the dynamic site index models with 2 site-specific parameters provide 2 important and desirable attributes of the site index, polymorphism and multiple asymptotes (Cieszewski, 2002).

Table 1. Some descriptive statistics for Scots pine and Oriental beech.

Species	Variables	Mean	Min.	Max.	Standard deviation	95% CI for mean
Pine	Tree ages	93.84	23	242	45.32	87.42–100.26
	Tree heights	19.53	6.8	35.1	6.79	18.57–20.49
Oriental Beech	Tree ages	87.94	23	162	31.26	83.56–92.31
	Tress heights	17.88	7.2	30.0	5.25	17.14–18.62



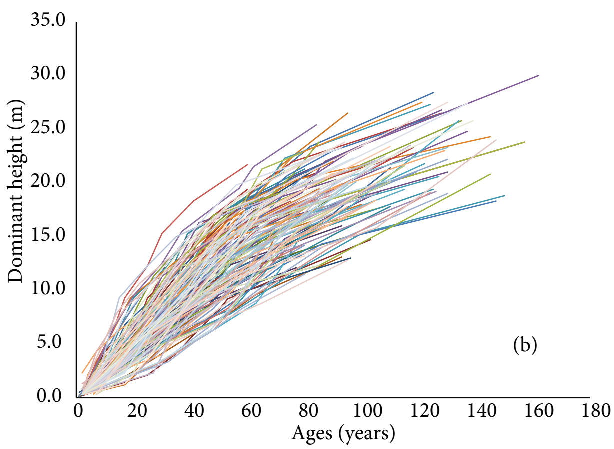


Figure 2. The profile plots of the stem analysis for Scots pine (a) and Oriental beech (b).

In the present study, 5 dynamic polymorphic models with multiple asymptotes, i.e. Bertalanffy–Richards (2 different forms for site-related parameters), Hossfeld (2 different forms for site-related parameters), and Lundqvist–Korf models, were selected as candidate functions to model dominant height growth for mixed Oriental beech (*Fagus orientalis* Lipsky) and Scots pine (*Pinus sylvestris* L.) stands. These dynamic structures of site index models were obtained by using GADA methodology (Cieszewski, 2002, 2004) derived from the base form of these growth models. These height growth models were distinguished from other growth functions to produce advanced polymorphic site equations. In particular, these 5 polymorphic algebraic difference models (M1–M5) were evaluated to satisfy various theories about growth characteristics, including multiple asymptotes, polymorphism, and other attributes. These dynamic site index models with 2 site-specific parameters depended on the relationships between theoretical site quality measure (*X*) and parameters of base growth models. This methodology for developing dynamic site models with 2 site-specific parameters is important for offering the opportunity to simulate dominant height–age relationships with polymorphic and multiple asymptotic growth models (Cieszewski, 2004). These polymorphism and multiple asymptote attributes are a crucial property of site index models (Cieszewski, 2002). The dynamic polymorphic model structures used to model dominant height–age relationships for mixed Oriental beech (*Fagus orientalis* Lipsky) and Scots pine (*Pinus sylvestris* L.) stands are presented in Table 2. These selected height–age functions have been commonly used to develop site index models (Corral-Rivas et al., 2004; Palahi et al., 2004; Diéguez-Aranda et al., 2005; Adame et al., 2006).

Nonlinear regression analysis was used to fit the dynamic base-age invariant site index models, nonlinear regression models M1–M5, in the present paper. This nonlinear regression analysis with the Marquardt algorithm for these functions was programmed using the SAS/ETS PROC MODEL procedure in SAS software for Windows (SAS Institute Inc., 2004).

Table 2. The dynamic GADA formulations with base models used to model dominant height growth.

No.	Base equation	Parameter related to site	Solution for X with initial values	Dynamic GADA formulation and reference
M1	Bertalanffy–Richards: $h=a(1-\exp(-bt))^c$	a=exp(X) $c=b_2+b_3/X$	$X_0 = \frac{1}{2}\left((\ln h_0 - b_2 L_0) + \sqrt{(\ln h_0 - b_2 L_0)^2 - 4b_3 L_0}\right)$ with $L_0=\ln(1-\exp(-b_1 t_0))$	$h = h_0\left(\frac{1 - \exp(-b_1 t)}{1 - \exp(-b_1 t_0)}\right)^{\left(b_2+\frac{b_3}{X_0}\right)}$ Cieszewski (2004)
M2		a=exp(X) c=b2+1/X	$X_0 = \frac{1}{2}\left((\ln h_0 - b_2 L_0) + \sqrt{(\ln h_0 - b_2 L_0)^2 - 4L_0}\right)$ with $L_0=\ln(1-\exp(-b_1 t_0))$	$h=\exp(X_0)$ $(1-\exp(-b_1 t))^{((b_2+1/X_0))}$ Cieszewski (2004)
M3	Hossfeld: $h = \frac{a}{1 + bt^{-c}}$	$a=b_1+X$ $b = \frac{b_2}{X}$	$X_0 = \frac{1}{2}\left(h_0 - b_1 - \sqrt{(h_0 - b_1)^2 - 4b_2 h_0 t_0^{-b_3}}\right)$	$h = \frac{(b_1 + X_0)}{1 + \frac{b_2}{X_0} t^{-b_3}}$ Cieszewski (2002)
M4		$a=b_1+X$ $b=b_2 X$	$X_0 = \frac{h_0 - b_1}{1 + b_2 h_0 t_0^{-b_3}}$	$h = \frac{b_1 + X_0}{1 + b_2 X_0 t^{-b_3}}$ Cieszewski (2002)
M5	Lundqvist: $h=a \cdot \exp(-bt^{(-c)})$	a=exp(X) $b = b_1 + \frac{b_2}{X}$	$X_0 = \frac{1}{2}\left(b_1 t_0^{-b_3} + \ln h_0 + L_0\right)$ with $L_0 = \sqrt{(b_1 t_0^{-b_3} + \ln h_0)^2 + 4b_2 t_0^{-b_3}}$	$h = \exp_{X_0} \exp :-\left(b_1 + \left(\frac{b_2}{X_0}\right)\right) t^{-b_3}$ Cieszewski (2004)

h and h_0 are dominant heights (m) at age t and t_0 (years), X is the parameter relating site, and a, b, c, b_1, b_2, and b_3 are fitted regression parameters.

2.2. Model selection and evaluation

The procedure used to compare and evaluate the site index models (M1–M5) was based on numeric and graphic analysis of the prediction and residuals of these models. This procedure has been widely used to compare and evaluate these models (e.g., Palahi et al., 2004; Adame et al., 2006; Martín-Benito et al., 2008). The first step included the comparison of the performance for the fitted height growth functions based on some evaluation criteria: bias ($\bar{E}$), root mean square error (RMSE), absolute mean error (AME), adjusted coefficient of determination (R^2_{adj}) and Akaike information criterion (AIC). In the second step, the behavior of model residuals was graphically examined to select the best predictive site index model. In the third step, the correspondence to desirable biological presumption for site index models was analyzed to provide efficient and accurate site index predictions, e.g., polymorphism, sigmoid growth pattern, multiple asymptotes, and passing a zero point. The model evaluation criteria were as follows.

$$\mathrm{Bias}(\bar{E}) = \sum_{i=1}^{n} \frac{h_i - \hat{h}_i}{n} \quad (1)$$

$$\text{Root mean square error (RMSE)} = \sqrt{\sum_{i=1}^{n} \frac{(h_i - \hat{h}_i)^2}{n - p}} \quad (2)$$

$$\text{Absolute mean error (AME)} = \sum_{i=1}^{n} \frac{|h_i - \hat{h}_i|}{n} \quad (3)$$

Adjusted coefficient of determination (R^2_{adj}) =

$$1 - \frac{\sum_{i=1}^{n} (h_i - \hat{h}_i)^2 (n - 1)}{\sum_{i=1}^{n} (h_i - \bar{h}_i)^2 (n - p)} \quad (4)$$

$$AIC = -2\log L + 2p \quad (5)$$

Here, h_i, $\bar{h}_i$, and $\hat{h}_i$ are the measured, average, and predicted values of the heights, respectively; is the maximum value of the log likelihood function; is the number of parameters to be estimated; n is the total number of observations; and is the number of model parameters.

3. Results

The models' parameter predictions, including its standard error, t-value, and P-value, and the models' goodness-of-fit statistics, including the R^2_{adj}, bias, RMSE, AIC, and AME, for Oriental beech and Scots pine are presented in Tables 3 and 4, respectively. Many parameters were found to be significant at $P < 0.05$, except for parameter b_1 in the M5 model for Oriental beech. These goodness-of-fit statistics calculated to 20-year age classes are presented in Tables 5 and 6. These goodness-of-fit statistics proposed that the M1 model, the Bertalanffy–Richards based on $a = \exp(X)$ and $c = b_2 + b_3 / X$ parameters relating to the site, had the best predictive ability with R^2_{adj} of 0.959, bias of 0.00171,

Table 3. Parameter estimations and goodness-of-fit statistics of candidate models for Oriental beech.

Model	R^2_{adj}	Bias	AIC	RMSE	AME	Parameter	Estimate	SE	t-value	Approx. P > t
M1	0.959	0.00171	723.55	1.353736	0.884034	b_1	0.01058	0.00055	19.23	<0.0001
						b_2	0.44181	0.2094	2.11	0.0351
						b_3	2.31358	0.7598	3.05	0.0024
M2	0.873	0.41685	1255.63	2.37774	1.77672	b_1	0.02714	0.00090	30.06	<0.0001
						b_2	1.18175	0.0696	16.97	<0.0001
M3	0.870	0.30013	1776.36	2.41041	1.81107	b_1	29.29525	0.3847	76.15	<0.0001
						b_2	−21,293	3471.4	−6.13	<0.0001
						b_3	2.24667	0.0439	51.22	<0.0001
M4	0.958	0.01046	760.52	1.37223	0.90175	b_1	55.67278	2.3878	23.32	<0.0001
						b_2	5.96×10^{62}	2.03×10^{-79}	294×10^{139}	<0.0001
						b_3	1.06277	0.0203	52.36	<0.0001
M5	0.959	0.03937	725.06	1.349514	0.886539	b_1	−4.10854	5.2776	−0.78	0.4364 ns
						b_2	66.66248	27.614	2.41	0.0159
						b_3	0.29339	0.0189	15.55	<0.0001

ns: nonsignificant.

Table 4. Parameter estimations and goodness-of-fit statistics of candidate models for Scots pine.

Model	R^2_{adj}	Bias	AIC	RMSE	AME	Parameter	Estimate	SE	t-value	Approx. P > t
M1	0.928	0.175911	1591.44	1.785801	1.216825	b_1	0.01934	0.00039	50.01	<0.0001
						b_2	−10.2060	0.11520	−88.59	<0.0001
						b_3	39.7305	0.4538	87.55	<0.0001
M2	0.736	0.828272	2775.06	2.650948	2.000875	b_1	0.03259	0.00137	23.78	<0.0001
						b_2	1.82438	0.0888	20.55	<0.0001
M3	0.707	-0.47088	2866.31	3.845965	3.059884	b_1	27.39699	0.4789	57.20	<0.0001
						b_2	−188.4810	2.5025	−75.32	<0.0001
						b_3	1.13426	0.0259	43.77	<0.0001
M4	0.951	0.049841	1250.78	1.55624	0.940128	b_1	44.84838	0.1131	396.44	<0.0001
						b_2	−46.4658	2.2410	−20.73	<0.0001
						b_3	1.50874	0.0154	97.72	<0.0001
M5	0.929	0.189858	1573.68	1.810775	1.132242	b_1	−550.614	29.2489	−18.83	<0.0001
						b_2	2158.184	113.8	18.96	<0.0001
						b_3	0.75666	0.0112	57.45	<0.0001

Table 5. The bias and RMSE values for different age classes for Oriental beech.

Age classes	Number of trees	Bias					RMSE				
		M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
21–40	78	−0.040	0.271	1.966	−0.109	0.140	0.505	0.582	2.465	0.529	0.530
41–60	117	0.071	0.142	1.226	−0.047	0.113	0.906	0.941	2.008	0.860	0.902
61–80	191	−0.046	−0.404	0.185	−0.132	−0.075	1.304	1.370	1.886	1.308	1.304
81–100	322	0.214	−0.582	0.160	0.189	0.219	1.253	1.450	1.944	1.273	1.227
101–120	160	−0.016	−1.311	−0.906	0.076	−0.050	1.581	2.348	2.516	1.590	1.577
121–140	183	−0.220	−2.638	−0.073	−0.278	−0.282	1.689	3.459	2.928	1.712	1.694
141–160	43	0.029	−2.087	−1.651	0.574	0.271	1.801	3.049	3.189	1.958	1.815
161–180	8	0.368	−2.436	5.347	0.505	0.806	0.849	3.158	5.868	0.931	1.110

RMSE of 1.353736, AME of 0.884034, and AIC of 723.55 as compared to the other models studied for Oriental beech. For Scots pine tree species, the M4 Hossfeld model based on $a = b_1 + X$ and $b = b_3 . X$ parameters relating to site had better predictive ability, with R^2_{adj} of 0.951, bias of 0.049841, RMSE of 1.55624, AME of 0.940128, and AIC of 1250.78, than the other models studied for Scots pine. The mathematical expression of the selected site index model for Oriental beech is as follows.

$$h = h_0 \left(\frac{1 - \exp(-0.01058_{t})}{1 - \exp(-0.01058_{t0})} \right)^{\left(0.44181 + 2.3135 \frac{8}{X_0}\right)} \tag{6}$$

$$X_0 = \frac{1}{2} \left((\ln h_0 - 0.44181_{L0}) + \sqrt{(\ln h_0 - 0.44181_{L0})^2 - 4_2.31358_{L0}} \right) \tag{7}$$

$$L_0 = \ln(1\text{-}\exp(-0.01058 \cdot t_0)) \tag{8}$$

Table 6. The bias and RMSE values for different age classes for Scots pine.

Age classes	Number of trees	Bias					RMSE				
		M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
21–40	144	0.071	0.536	−0.197	0.188	0.643	0.579	0.882	2.221	0.619	0.937
41–60	133	0.275	0.567	−0.716	0.284	0.607	0.809	0.960	2.700	0.750	1.028
61–80	163	0.391	−0.048	−1.342	0.081	0.269	1.345	1.267	2.712	1.200	1.290
81–100	143	0.650	0.369	−0.260	0.555	0.479	1.802	1.259	3.319	1.334	1.545
101–120	333	0.434	−0.449	−0.613	0.172	0.172	1.861	1.564	2.628	1.400	1.685
121–140	175	−0.777	−1.753	−0.028	−0.811	−0.898	2.533	3.161	1.622	2.330	2.560
141–160	47	−0.375	−1.585	0.732	−0.274	−0.333	1.814	2.744	1.562	1.728	1.746
161–180	34	0.879	−1.487	−0.165	0.674	1.124	1.957	2.526	1.442	1.688	2.033
181–200	27	−0.280	−2.601	−0.038	−0.169	0.421	1.630	3.468	1.024	1.477	1.578
201–220	8	1.606	−2.104	−0.488	1.427	2.357	2.507	2.992	1.273	2.580	3.344
221–240	9	0.205	−2.676	−0.626	0.489	1.734	1.500	3.864	0.712	1.066	2.146

The forms of the selected site index model for Scots pine are as follow.

$$h = \frac{44.84838 + X_0}{1 - 46.4658_{X0} t^{-1.50874}} \quad (9)$$

$$X_0 = \frac{h_0 - 44.84838}{1 - 46.4658_{h0} t_0^{-1.50874}} \quad (10)$$

Here, is the dominant height at age of t, and h_0 is the dominant height (site index, i.e. 10, 12, 14, 16, 18, 20, or 30 site index values) at age t_0 (base-age, i.e. 30, 50, or 100 years).

In Figure 3, the behaviors of residuals for predicted heights by these best predictive site index models based on Model 1 (Figure 3a) for Oriental beech and Model 4 (Figure 3b) for Scots pine are graphically presented. These site index models provided random outlines of residuals around zero and no obvious significant trends (Figure 3). Thus, there is no serious breakdown of homoscedasticity, violations of the assumption of constant variance, in these models for predicting site index values. The results of the F-test (Figures 4a and 4b) showed that there no was reason to reject the null-hypothesis of intercept = 0 and slope = 1, meaning that there were no systematic over- or underestimates in site index models for Scots pine (F-value = 33749.2, P < 0.01) or oriental beech (F-value = 25779.3, P < 0.01). These desirable characteristics of the residual pattern underlined the statistical acceptability of these models as statistical regression models with no bias prediction results.

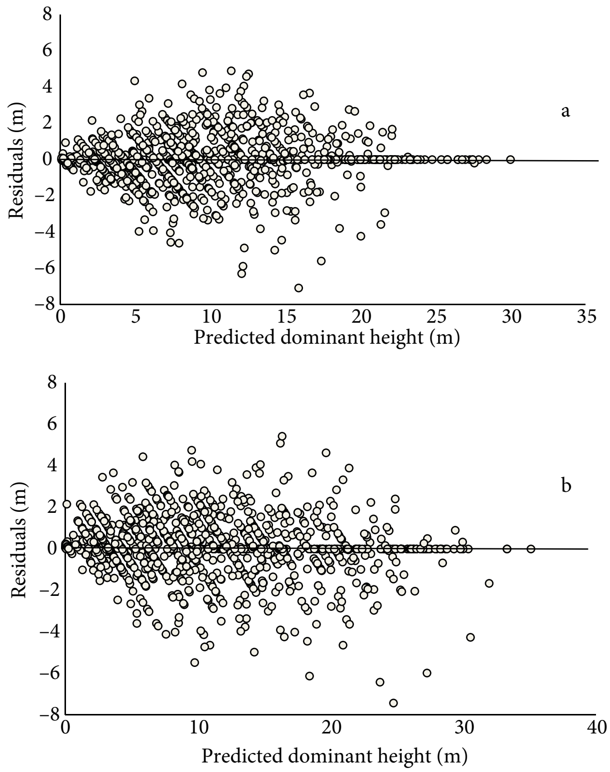


Figure 3. Residuals against predicted dominant height to predicted height for Model 1 fitted with Oriental beech trees (a) and Model 4 fitted with Scots pine trees.

In addition to evaluating residuals for the best models selected, the desirable biological presumptions of site index models, i.e. polymorphism, sigmoid growth pattern,

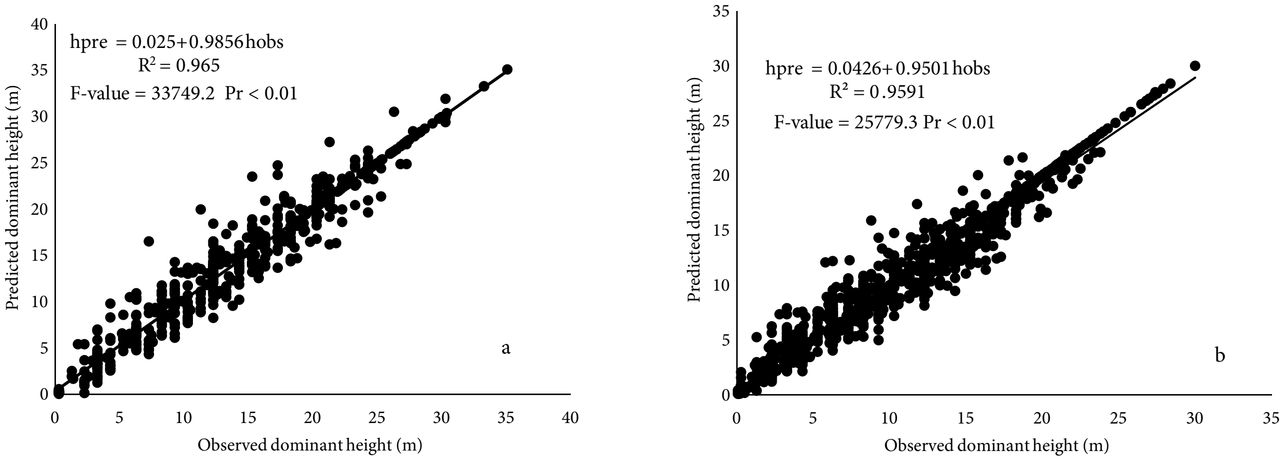


Figure 4. Predicted dominant height against observed dominant height for Model 4 fitted with Scots pine trees (a) and for Model 1 fitted with Oriental beech trees (b).

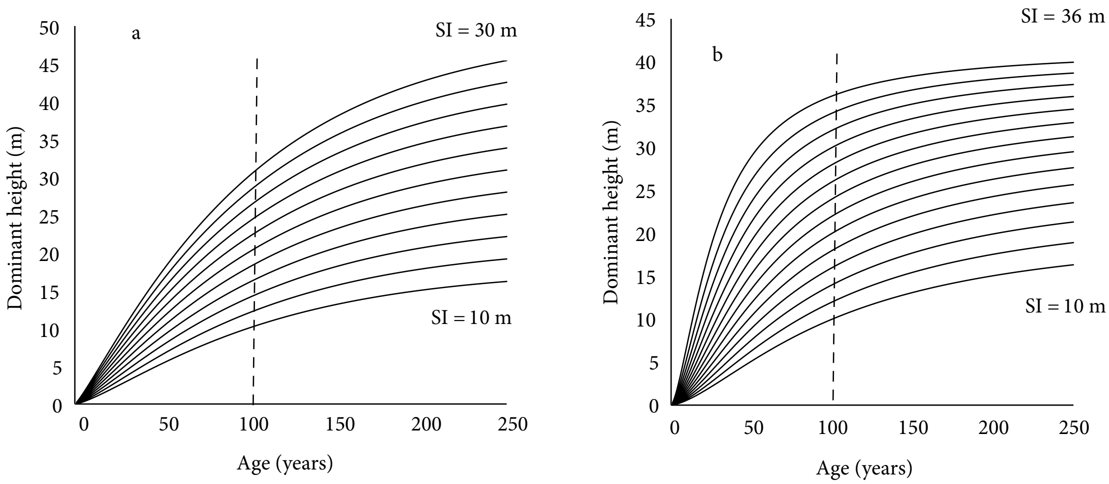


Figure 5. The site index curves generated with Eq. (1) for Oriental beech (a) and Eq. (4) for Scots pine (b) to 250 years.

and multiple asymptotes, were checked as important attributes for developing site index prediction systems. In this regard, the families of site index curves with different site indices (SIs), including 10, 12, 14 …. 26, 28, and 30 m at 100 years for Oriental beech and 10, 12, 14 …. 30, 32, 34, and 36 m at 100 years for Scots pine, are presented in Figure 5a for Oriental beech and in Figure 5b for Scots pine. Initially, the shape of these site index curves fulfilled some desirable biological presumptions for site index predictions, i.e. polymorphism and sigmoid growth pattern. When these graphs are examined, it is seen that a polymorphic sigmoid growth pattern, which had different shapes at each site index value, was accomplished for M1 for Oriental beech and M4 for Scots pine. Another desirable property, multiple asymptotes, which have been implied as the varying asymptotic values of models with site index values, can be perceived with some simulations of height predictions for 500 ages for these tree species in Figures 6a and 6b.

In Figure 7, the mean annual increments, , of height predictions by GADA models for Oriental beech and Scots pine are presented to examine the average height growth trends per year. When these graphs are examined, the MAIs of height started with small values and then increased to a maximum value as the trees matured and declined slowly at older ages. By analyzing the graph, the relationship between ages for the peak point of these curves, also called the inflection point, and site index values can be determined for Oriental beech and Scots pine. In these relationships, the ages at the time of reaching the inflection point were decreasing with site index values; however, the values of MAI at the inflection point were

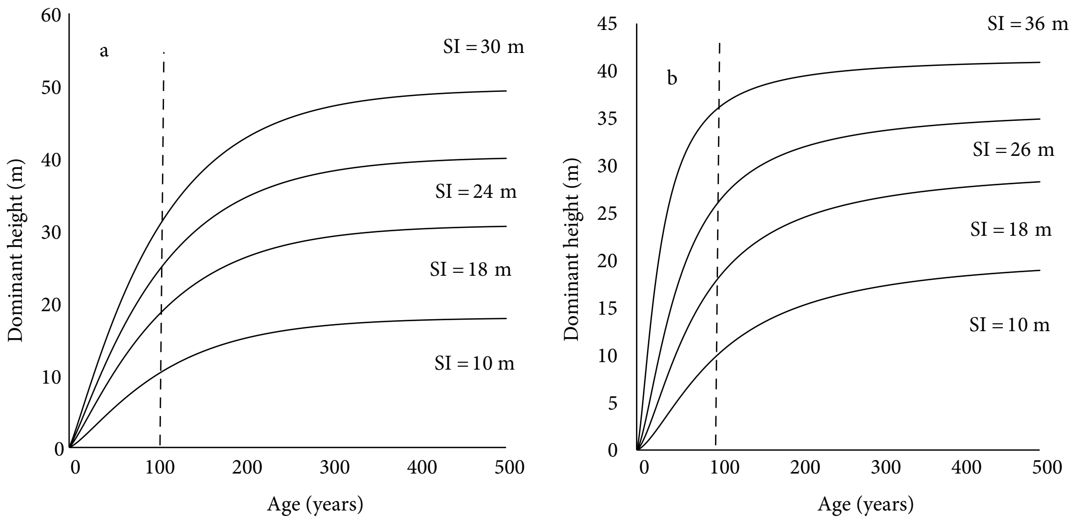


Figure 6. The site index curves generated Eq. (1) for Oriental beech (a) and Eq. (4) for Scots pine (b) to 500 years.

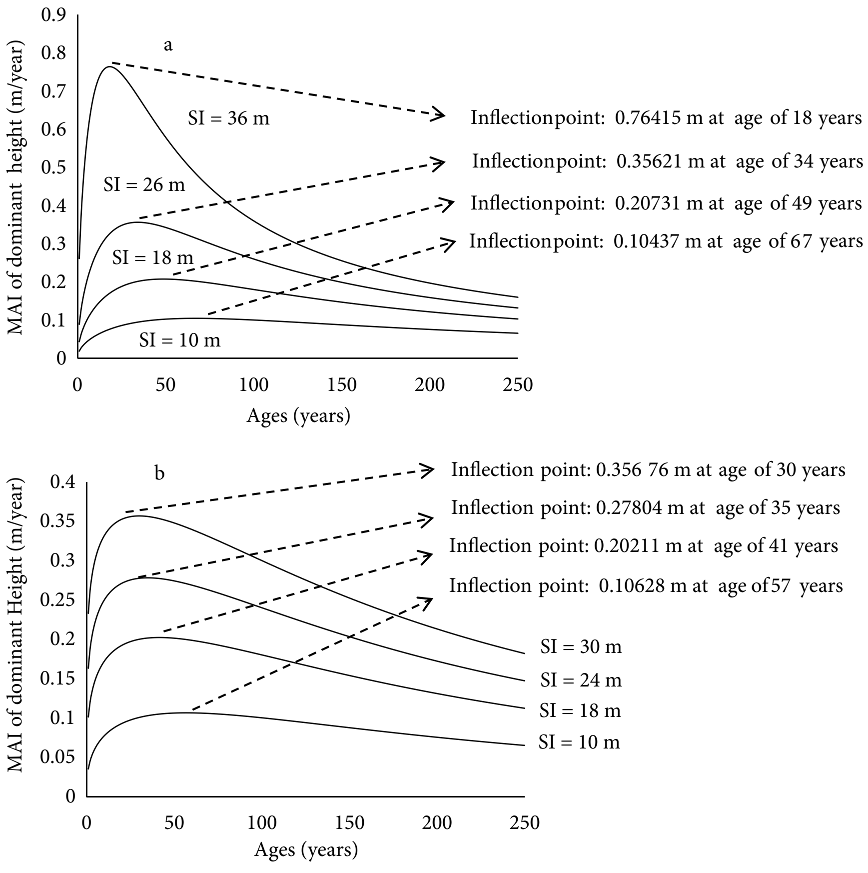


Figure 7. The mean annual increments calculated based on Eq. (1) for Scots pine (a) and Eq. (4) for Oriental beech (b).

increasing with the greater site index values for Oriental beech and Scots pine. For example, the inflection point of the best site, SI = 30 m, had an MAI of 0.35676 m at an age of 30 years, and the poorest site, SI = 10 m, had an MAI of 0.10628 at an age of 57 years for Oriental beech. In Scots pine, the best site, SI = 36 m, had an MAI of 0.76415 m at an age of 18 years, and the poorest site, SI = 10 m, had an MAI of 0.10437 at an age of 67 years.

In Figures 8 and 9, previous site index curves obtained by Carus (1998) for Oriental beech and by Alemdağ (1967) for Scots pine are compared with the new site curves developed for mixed Oriental beech and Scots pine stands in the present study. Noticeably, it can be seen that there were important differences between previous site index curves and the new site index models developed with the GADA. The site curves developed by Carus (1998) for Oriental beech were higher than the new site curves until an age of about 100 and were lower for older ages. Alemdağ's site curves (1967) were fairly equivalent to the new site curves for the poorest (SI = 10 m) and moderate (SI = 20 m) site qualities, but the differences between earlier site curves and new site curves were obvious for the best site qualities (SI = 30 m) for Scots pine. It can be seen from Figures 8 and 9 that previous site index curves absolutely underestimated dominant height at older ages for Oriental beech and until an age of 100 for the best sites of Scots pine. In Figures 10 and 11, graphical comparisons are presented for the new site curves developed by the GADA in this study, the Nord-Larsen (2006) model for European beech, and the Palahi et al. (2002) model for Scots pine. When these graphs are examined, important differences can be seen for dominant height growth trends of these Oriental beech and Scots pine species located in the northwest of Turkey as compared to those growing in forest areas of central Europe, i.e. European beech and Scots pine stands.

4. Discussion

This study presents dynamic base-age invariant site index models based on the GADA for mixed Scots pine (*Pinus sylvestris* L.) and Oriental beech (*Fagus orientalis* Lipsky) stands. The site index models, based on Bertalanffy–Richards, Hossfeld, and Lundqvist–Korf growth functions, were fitted for these mixed stands. These site index models were compared by numeric and graphic analysis and

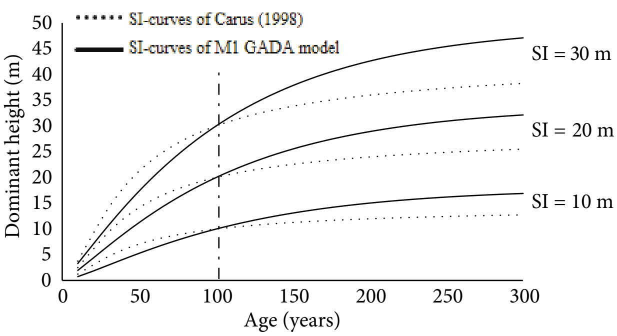


Figure 8. Site curve comparisons obtained from new site index model based on GADA and Carus' (1998) earlier model for Oriental beech.

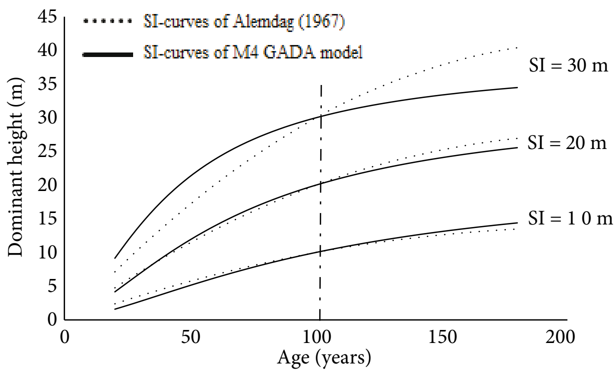


Figure 9. Site curve comparisons obtained from new site index model based on GADA and Alemdağ's (1967) earlier model for Scots pine.

Figure 10. Site curve comparisons obtained from GADA developed in this study for Oriental beech and Nord-Larsen's (2006) model for European beech.

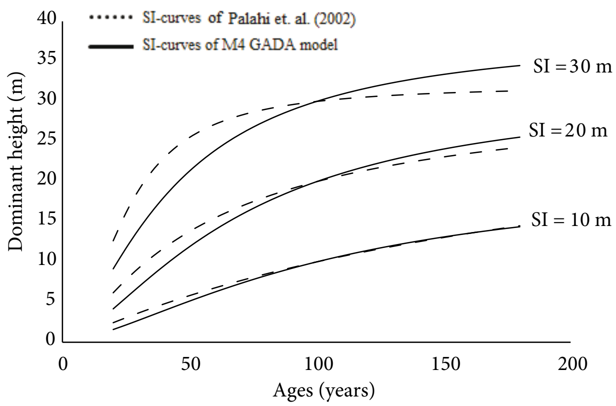


Figure 11. Site curve comparisons obtained from GADA developed in this study and the model of Palahi et al. (2002), both for Scots pine.

then the best selected models were further analyzed for the desirable attributes and biological presumptions for site index predictions. Among the 5 different dynamic site index equations, the Bertalanffy–Richards M1 model for Oriental beech and the Hossfeld M4 model for Scots pine showed the best predictive abilities based on some fit statistics, i.e. , bias, RMSE, AME, and AIC. The best predictive dynamic site index models, based on M1 for Oriental beech and M4 for Scots pine, accounted for 95%–96% of the total variance in height–age relationships in dominant trees.

The dynamic polymorphic site index models with multiple asymptotes are more successful in fulfilling all of desirable properties of height growth patterns than either anamorphic or single polymorphic models, i.e. earlier site index models. The achievement of GADA-derived dynamic site index models in being able to describe the height growth pattern based on both biological and statistical aspects has been reported in the findings of many other studies, such as those of Cieszewski and Bailey (2000), Cieszewski (2002, 2003, 2004), Corral-Rivas et al. (2004), Diéguez-Aranda et al. (2005, 2006), and Martín-Benito et al. (2008).

These best predictive site index models have an advantage in their ability to produce invariant predictions ensuring different "base-age" projections. The fixed-base-age site equations require an arbitrary choice of base-age prior to fitting equations. Thus, the site index predictions attained by these models highly depend on the selection of "base-age" at the beginning of the site equations' model development stage. Thus, these fixed-base-age site index models cannot satisfy effective and successful site index predictions for some situations occurring in different "base-age" selections. The dynamic site index models based on GADA derivations using the Bertalanffy–Richards equation for Oriental beech and the Hossfeld equation for Scots pine present base-age invariant site index predictions by incorporating t_0 variables into the model's structure. Consequently, the base-age invariant site index presents dynamic predictions with different "base-ages". In addition, these dynamic base-age invariant site index models can provide some additional desirable attributes regarding model predictions: 1) sigmoid growth pattern with an inflexion point, 2) defined origin at time equal to zero, and 3) heights at base-ages equal for site indices with different base-ages.

The site index models developed in this study are polymorphic dynamic site curves with multiple asymptotes; however, earlier site index models developed by Carus (1998) for Oriental beech and Alemdağ (1967) for Scots pine are anamorphic with single asymptotes. It is important to note that the previous site index curves developed by Carus (1998) for Oriental beech and Alemdağ (1967) for Scots pine were developed to model dominant height growth by anamorphic height functions for pure stands of these species. These attributes are reasons for the large differences in height growth trends between previous site curves and new dynamic site index curves.

The developed dynamic base-age invariant site index models can be expected to give satisfactory predictions with operative and enhanced predictive results with the GADA methodology for mixed Scots pine (*Pinus sylvestris* L.) and Oriental beech (*Fagus orientalis* Lipsky) stands. However, the techniques of site index models including proportions of species for mixed stands can improve the predictions of site index in forest inventory for these mixed stands. In this regard, the effect of admixture in dominant growth with interspecies interactions should be given special importance in forestry research. In particular, new practitioner modeling techniques including interspecies interactions and mixed-stand dynamics, e.g., competition reduction and facilitation, are required to develop these site index modeling systems for mixed stands at the different levels of tree species proportions. Thus, innovative dynamic site index models may provide better estimating of the natural height growth trend with species interactions as distinct from those of pure stands than classical site index modeling techniques.

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