SERD

The Investigation of Middle School Students' Misconceptions about Algebraic Equations*

Nihan Uçar Sarımanoğlu[1]

Received: 31. 12. 2018 **Accepted:** 25. 06. 2019

How to cite: Uçar Sarımanoğlu, N. (2019). The Investigation of Middle School Students' Misconceptions about Algebraic Equations. *Studies in Educational Research and Development, 3*(1), 93-114.

Abstract

This qualitative study focused on the errors and misconceptions of the seventh grade students in their solutions for algebraic equations. For this aim, a question sheet including 17 algebraic expressions were implemented to 10 seventh grade middle school students in a middle school of Ankara. Semi-structured interviews were also conducted with two participants. After administering the question sheets, the answers of the students were categorized by coding the misconceptions of the students. Then the written and recorded answers were organized, coded, categorized and discussed. The results showed that, mostly observed misconceptions among the participants' solutions for the algebraic expressions were (a) inability of making operations between the variables based on the lack of knowledge about arithmetical operations, (b) believing that the variables are used only for the natural numbers, and (c) ignoring the letters included in the algebraic expressions.

Keywords: Algebra, misconception, equations.

Introduction

Algebra has an undeniable importance in mathematics. In the schools, students are introduced with arithmetic from the first class, so they subtract, add, multiply, and divide numbers (MoNE, 2005). They come to face with word problems and make calculations offered in these word problems. However; when the calculations are offered as algebraic expressions, some problems may occur (Jupri & Drijvers, 2016). The calculations are still named as subtraction, addition, multiplication, and division but sometimes it is observed that students cannot be serial in doing operations as

---

* This study was presented in the conference named as European Conference on Educational Research (ECER, 2015).

[1] ORCID: 0000-0001-7600-5957, Dr., Ministry of National Education, General Directorate of Innovation and Educational Technologies, Ankara, Turkey, nihanucar@ymail.com

they are in arithmetical expressions. Besides, their emphasis on computation leads to many misconceptions in students' minds, which in turn makes the learning of algebra become more difficult (Baroudi, 2006).

It can be thought that learning algebra is the next step which is coming after arithmetic and students learn the meanings of arithmetical expressions easier than algebraic expressions. However according to Romberg, Carpenter, and Kwako (2005), students do not realize that the procedures in solving the equations and simplification of the expressions are based on the same properties that have been used in arithmetic. So; when they are told that the operation rules are similar with the ones valid for arithmetic, the students become more comfortable with the algebraic operations.

The studies showed that students have difficulties to understand the meaning of a variable in an algebraic equation (Davidenko, 1997; English & Warren, 1998; Knuth, Aibali, McNeil, Weinberg, Madison, 2005; Küchemann, 1978; Macgregor & Satcey, 1997; Philipp, 1992; Wagner, 1983). Misunderstanding of algebraic calculations, on the other hand, may appear because of the misunderstanding of the variables (Christou, Vasniadou & Vamvakoussi, 2007). For instance; students may name the literal symbols as labels for objects (Christou, Vosniadou & Vamvakoussi, 2007). They think the y in ''add 3 to y'' is anything like yogurt or yum, or they think that the letter such as D refers to a name like David. These kind of misconceptions make the questions seem unsolvable for the students.

The idea that algebra can and should contribute constructively to the curriculum is a phenomenon in schools. Romberg, Carpenter, and Kwako (2005) claim that, engaging students in algebraic thinking and making this thinking visible to the teachers are very important issues. In Turkey, the algebraic thinking is expected to be gained in early grades (MoNE, 2005). Therefore, for realizing the positive potential of teaching algebra in classrooms, a student needs a clear idea of what he/she is dealing with during the operations in algebraic expressions. Moreover; mathematics teachers need a clear idea of what the misconceptions of the students are about their algebraic operations. Determination of the misconceptions may be helpful for their teaching and assessment processes. The lack of knowledge coming from the background of the students, or the teaching processes can be identified with fixing the misconceptions done in algebraic operations (MoNE, 2005).

Foster (2007) claims that teachers should understand the sources of the misconceptions of the students in order to construct a permanent teaching environment. Moreover; Erbaş and Ersoy (2003) claim that the misconceptions appeared in the first steps of the algebra courses should be identified correctly and education of the students should be designed in the aim of dealing with these misconceptions. Thus; revealing the misconceptions of the students is important for the teachers, curriculum developers, and the students.

Review of Related Literature

Algebra takes place in early times of the school years. Students introduce with the algebraic expressions with the word problems or with the unknown number included exercises. There are lots of studies showing that students' misconceptions of algebra are mostly related with their lack of knowledge about the arithmetical operations (Mulungye, 2010; Norton & Irvin 2007; Stacey & Chick, 2004; Stacey & Macgregor, 1999; Wu, 2001). Besides, it is showed that middle school students have misconceptions mostly about the meaning of equal sign (Behr, Erlwanger, & Nichols, 1980; Falkner, Levi, & Carpenter, 1999; Kiearan, 1992; Cheng-Yao, Yi-Yi, & Yu-Chun, 2014).

For instance, Baroudi (2006) made a study that is concerning two categories of misconceptions: related with the equivalence concept and related with doing four operations. The interviews with the participant students showed that, they were aware of the rules such as 'a division operation is not commutative'. However, they did mistakes in solving paper-and-pencil questions. For instance; they wrote that 3:5 = 5:3 or they thought that division of two whole numbers gives a whole number. Baroudi (2006) claims that, the reason for such contradictions are based on the inadequate variety in computation practices that students had in their academic backgrounds.

Gonzales, Ambrose and Martinez (2004) also conducted a study in order to analyze the understandings of the equal sign. They studied with two groups. One of the group was composed of fifteen 3rd grade students whereas the other group was composed of 26 students with 5th and 6th grades. It was understood by the study that, the first group (3rd graders) could not explain the meaning of the equal sign. On the other hand, students could not recognize the relational thinking while they were answering the questions like 15 + 4 = 4 + …… The second group (5th and 6th graders), however gave right answers. It was told by the researchers that, only the subtraction operation seemed to be difficult and answered wrongly. The researchers advocated that this should be because of the relational thinking inabilities of the students during the subtraction operations.

Different from the study conducted by Gonzalez, Ambrose, and Martinez (2004) and Baroudi (2006); Vendlinski, Howard, Hemberg, Vinyard, Martel, Kyriacou, Casper, Chai, Phelan, and Baker (2008) constructed a study in order to investigate whether the teacher effectiveness is a controllable variable in improving students' achievement in algebraic operations. It was shown by the study that, representations including graphical arrows like 2 (x + 4) cause some misconceptions in algebra. For instance; students may think that such a distribution can be possible across multiplication and division, too. Moreover, without a complete understanding of distribution, first-year algebra students are also think that, they can distribute exponents over addition so that $(x + 2)^2$ becomes $x^2 + 2^2$ (Vendlinski et al., 2008).

Akkaya and Durmuş (2006) conducted a study in order to understand the misconceptions of the students of 6-8 grades in their algebra practices. The sample of the study consisted of 280 students from 15 primary schools of Bolu. The students were at grade 6-8. A multiple choice test was implemented to the participants. The results of the study showed that students had variety of misconceptions about the meaning of the letters in algebra. For instance they thought that, the order of the letters in the alphabet is similar with the order of the numbers. For example, since the letter "c" was the third letter in the alphabet, most of the students thought that "c" had the value of 3 in algebraic problems. The students also thought that, letters used instead of the unknowns had only digit values. For example, "ab" can be interpreted as a two-digit number. Therefore; students did not think that the expression like ''ab'' was the multiplication of the numbers ''a'' and ''b''. Third, the order of the operations and the importance of the parentheses in the questions were not taken into consideration by the participants. Another misconception found in this study was the thoughts of the participants about letters expressed in the algebraic expressions. For instance, the term "3a" is interpreted only as three times the numbers of apples. Therefore the letter "a" is for the word "apple" whereas "a" can be used for any unknown in algebra.

Matzin and Shahrill (2015) conducted a study with 78 seventh grade students. In their study they tried to figure out which area of solving algebra problems was the most difficult part according to the students. Moreover they investigated how much algebraic knowledge from their past mathematics studies was remembered by seventh grades. Similar with the results of Akkaya and Durmuş (2006), Matzin and Shahrill (2015) observed that most of the participants had problems in formulating and manipulating the algebraic equations correctly.

In their study with 50 seventh grade students, Şahin and Soylu (2011) also observed that, students attributed to the variables as they are only digit values. They perceived the variables in a multiplication operation as if they were digits, not a multiplier. In the study it was also interpreted that, 'ignoring the presence of letters in an algebraic equation' is the most common misconception among the participants.

Misconceptions related with variables (the letters labeling the variables) in algebraic equations or expressions are widely studied. In lots of such studies it is observed that, one of the common misconception in algebraic expressions is comprehending the letters as objects or labels (Asquith, Stephens, Knuth, & Alibali, 2007; Clement, 1982; MacGregor & Stacey, 1997; McNeil, Weinberg, Hattikudur, Stephens, Asquith, Knuth, & Alibali, 2010; Usiskin, 1988).

In some studies it is also figured out that, some students ignore the presence of variables. For instance when they are asked to solve (n+5+4), 20 % of the students answered the question as 9, ignoring the presence of n (Booth, McGinn, Barbieri, & Young, 2017). Most of them also believe that, the letter is associated with its order in the alphabet (Asquith et al., 2007; MacGregor & Stacey, 1997).

Another type of misconception affecting students of all levels is about the use of brackets and the order of operations in algebraic equations (Booth et al., 2017). In that manner, many students do not need to adhere to the order of operation rules and continue to solve the expressions from left to right (Gardella, 2009). Additionally, many students fail to realize that brackets can be used to both grouping and signal multiplication (i.e. (20 – 7) = 13 and – (20 – 7) = -13) (Booth et al., 2017).

National Curriculum Council for Great Britain (1992) summarized the types of misconceptions (see Table 1.) that the students had while they solved the algebraic exercises (NCC, 1992).

Table 1. List of misconceptions in algebraic equations

Ignoring the presence of the letters
Not taking the letters as variables (e.g.: comprehending the letters as objects)
Thinking that letters always have one specific value
Thinking that the letters can only stand for the natural numbers
Treating to the operation symbols as they are not exist in an expression
Ignoring to use the rules for brackets in an algebraic expression
Inability of making operations between the variables based on the lack of knowledge about the arithmetical operations
Taking no notice for the negative/positive signs while manipulating the algebraic expressions

Having misconceptions in any mathematical field may create disadvantages for successful problem solving or for learning new information (Chi, 1978; Tenenbaum, Tehan, Stewart, & Christensen, 1999). Booth and Koedinger (2008) conducted a study to investigate how having misconceptions affected students' ability to solve algebraic equations correctly and to learn correct problem solving procedures. For the study, 49 high school students who were learning to solve simple equations using the cognitive tutor curriculum completed a pretest-posttest design in order to evaluate their conceptual understandings about negative and positive signs. In the cognitive tutor curriculum, students learn various kinds of representations for solving algebra problems (Koedinger, Anderson, Hadley, & Mark, 1997). The participants did not know the solution of two-step linear equations but could complete the course including prior problem solving methods. The study showed that students who had misconceptions about the meaning of the equals sign or negative sign concepts, solved fewer equations correctly at pretest, and also had difficulties in learning how to solve them.

Having misconceptions about the negative/positive signs might persist into the university years. In their study with university students, Cangelosi, Madrid, Cooper, Olsen, and Hartter (2013) found an example result for this claim. They observed that, university students had difficulties in manipulating exponential expressions which included a negative sign as a part of the base or exponent.

In accordance with the related research above, the current study aimed to understand the students' misconceptions while they were solving algebraic equations. In the light of the study, it was aimed to make positive contribution to the algebra lessons by representing the misconceptions of the 7th grade students about their practices in algebraic equations. Therefore; the research question of the study was as follows: 'What are the misconceptions of the participant seventh grade students in their algebraic equation practices? '

Methodology

In order to answer the research question, the qualitative approach was used in the study. It was conducted with 10 students from seventh grade. Convenient sampling was used as the sampling method of the sample. It was convenient in terms of the ease of access and ease of getting official permission from school administration. The participants were from a middle school having 40 seventh grade students and they were accessible to reach. 7 students were boys and 3 students were girls. 4 of the students were high-achieved students in mathematics, 2 of them low-achieved in mathematics and 4 of the participants were medium-achieved in mathematics. The achievement degrees were identified by the academic performance of the students in the mathematics lessons in the current academic year.

Instrument

The starting point of the instrument of the study was based on the one used by Akkaya and Durmuş (2006) in their study about the misconceptions of the middle school students. In order to identify the misconceptions of the participant students, the questions of that instrument were turned out from multiple-choice-questions to open-ended-questions. There were 17 questions in the question sheet. All of the questions were about algebra. The questions were modified to be suitable for the 7th grades.

The mathematics curriculum offered by the Ministry of National Education (2009) was used as a rationale for making the questions suitable for the 7th grade students. The instrument constructed by Akkaya and Durmuş (2006) was also the other measure and was taken as reference to supply the criterion-related evidence. The instrument of that study was constructed for administration to 6th, 7th, and 8th grade students. The questions were modified to be suitable for the 7th grade students. Thus; in order to satisfy the validity of the instrument, the questions were discussed with another mathematics teacher working in another middle school before the administration. There was not a mathematics teacher other than the researcher in the current school. Therefore; the expert opinion was gotten from a teacher working in another school. Thus; it can be said that a combination of the literature review and the expert view considered while the items for the instrument were constructed.

After the expert opinion, a pilot study was done with 10 seventh grade students in another school. After the pilot study, the changes took place. The number of the questions was 25 in the pilot study. However; it was understood that some questions were not clear. Therefore; some changes were done in the type and the number of the questions in the instrument. A pilot study was also constructed for the semi-structured interviews. 2 students of the pilot study participants were the volunteers. However; nothing was changed in the way of the interviews after the pilot study. The questions that were used to collect data were listed in Table 2.

It took nearly 30 minutes for the students to answer the questions of the main study. In order to identify the solution methods and the misconceptions of the students, semi-structured interviews were also constructed with the 2 volunteers from the participants of the study. They were informal semi-structured interviews. The interviews took about 10 minutes for each of the students and a tape-recorder was used during the interviews. Note-taking was also used during the interviews.

Table 2. The list of the questions that were used for collecting data

Number of the Question (Q)	Question
1st	2a + 8 = 10. Find the value for a
2nd	2x + 3x + 30 = 40. Then what is the value of x?
3rd	x/4 + 10 = 80. Find the value of x.
4th	2y + 5y + 3 = 10. What is the value of y?
5th	x/3 + 2x/3 = 10. Then what is the value of x?
6th	2y + 8 = y + 10. Then what is the value of y?
7th	3x + 6x = 18. Find the value of x
8th	x/5 + x + 3x/10 = 32. Then what is the value of x?
9th	x + y + z = 30. Then find the values of unknowns in the equation
10th	Find the value of of 3x + 8, if x = 2
11th	Express the statement "3 less than 2 more of a number" as an algebraic form.
12th	3(2x + 9)/5 = 9. Then what is the value of x?
13rd	3(1 – 2m) = 4m – 5. Then what is the value of x?
14th	Find the number of unknowns in the following algebraic expression: X + m + 8 = 30
15th	2(2a – 9) – 7a + 6 = 0. Then what is the value of a?
16th	1 – 8k = 0,2. Then what is the value of k?
17th	9 – 7x = 3. Then what is the value of x?

Procedure

The research investigated the misconceptions of the seventh grade students in their algebra practices. For this aim, a question sheet with 17 questions was administered to 10 seventh grade students. After the implementation, an interview was made with 2 volunteered participants.

The participants of the study were students at the same school. Thus; the permission for the study was taken from the school administration. Then, the question sheets were administered to the seventh grade students who were all students in the same middle school of Mamak district in Ankara. This district was chosen since the implementation would be convenient in terms of communication with the teachers.

The questions were answered in approximately 30 minutes in one session in one classroom. To prevent the study become invalid, some other precautions also were supplied. First of all; the question sheets were administered to the participants and the researcher stayed in the classroom as an observer. Second; in order to prevent them from cheating, the aim of the question sheet was told to the participants. It was clearly identified that, the answers of these questions would not affect their routine performances in the classrooms. They were informed that the results would not have a quantitative worth on their semester performances. Third; it was asked from the students to solve the questions themselves. To prevent the study become invalid because of cheating, each student was alone in a desk during the implementation. The researcher noted the verbal questions of the students during the implementation. Last of all; member checking was done to construct validity. For member checking, some answers of the participants were asked to them again as a confirmation. With the help of member checking it was aimed to reduce the researcher bias threat, the respondent bias threat, and reactivity threat of the validity (Frankel & Wallen, 2003).

Lastly, the semi-structured interviews were conducted with approximately in 20 minutes (10 minutes per student). In the interviews, it was asked the students to explain their ways of the solutions. It was aimed to understand the misconceptions that caused the wrong answers.

Limitations

First of all, convenience sampling may be a limitation for the study. Second, this study is limited to the participants' understandings of the questions mentioned both in the question sheets and in the interviews. None of them had an experience about such a research so they might be nervous or excited during the study.

The findings of the current study are limited in terms of generalizability. In other words, the findings were limited with the current study and were evaluated in the contexts of the mathematics curriculum and related literature.

Lastly; although she implemented the steps for providing reliability, the researcher's bias might have limited the current study. She collected the data individually which may also have limited the study. For instance, she had to record and take notes at the same time. This might cause focus problem, so some data might be overlooked. Moreover, the findings were explained according to researcher's interpretations. She was the observer, the interviewer, and analyzer during the study. Besides, she was a teacher in that school. As a result of that, the study might have been affected by the researcher's views and teaching experiences.

Analysis of Data

After administering the question sheets, the answers of the students were categorized. Data of the study were collected in a less-structured, open-ended format (Frankel & Wallen, 2003) and the categorization was made by coding the misconceptions of the students. Then the written and recorded answers were organized, coded, categorized and discussed.

For the data analysis, a coding sheet based on the misconception list of NCC (1992) was used. There were the names of the misconceptions in that list. The answers of the participants were coded in the suitable places of the list. The coding operation was supported by the interviews in order to give significant meanings to the answers of the students. In addition to the data gotten by note taking, the interviews were recorded in to a tape recorder. Therefore; the words recorded or the notes taken during the interviews and administrations were helpful to fix the codes. The frequency distributions of the coded results were constructed in a tabular form.

Results

In Table 3, the misconceptions determined by The National Curriculum Council for Great Britain (NCC, 1992), the questions in which the students' answers/solutions showed misconception and the number of the participants who did the misconception are represented.

Table 3. Misconceptions and the distribution of the misconceptions

Misconception	The question by which misconception is observed	Number of the participants
Ignoring the presence of the letters	1st, 2nd, 7th, 14th	3
Not taking the letters as variables (e.g.: comprehending the letters as objects)	16th	1
Thinking that letters always have one specific value	-	-
Thinking that the letters can only stand for the natural numbers	9th, 5th	4
Treating to the operation symbols as they are not exist in an expression	1st, 2nd, 3rd, 10th	2
Ignoring to use the rules for brackets in an algebraic expression	13th, 15th	1
Inability of making operations between the variables based on the lack of knowledge about the arithmetical operations	11th, 15th, 16th, 17th	6
Taking no notice for the negative/ positive signs while manipulating the algebraic expressions	15th	2

It can be seen by Table 3 that, six participants showed inability of making operations between the variables based on the lack of knowledge about the arithmetical operations. Behaving as if ‘the letters are used only for the natural numbers’ is another misconception observed in four participants’ answers. On the other hand, it can be seen by the table that, there are not any observed misconception about ‘thinking that letters always have one specific value’ in the participants’ answers. In the following section, example photos of the participants’ algebraic equation misconceptions that are identified in Table 3 are given.

Examples from the Participants' Question Sheets by Their Handwritings

Three of the participants 'ignored the presence of the letters'. Figure 1 is an example photo showing the current misconception which is observed in a participant's answers for 1st and 2nd questions:

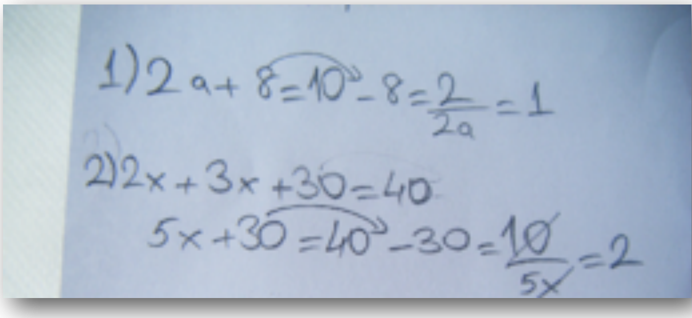

Figure 1. An example for 'ignoring the presence of letters'

It can be seen in Figure 1 that, during the solution of 1st question, the participant student ignores the letter "a" and divides "2" by "2a" and finds the solution as 1. The answer is correct; however, there seems to be error in the operation part. The same misconception exists in the solution of 2nd question, too.

'Taking no notice for the negative/positive signs while manipulating the algebraic expressions', is another misconception observed in the study. Figure 2 represents an example for this misconception from a participant's answer for the 6th question:

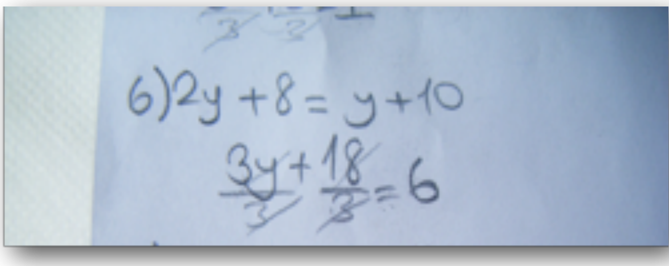

Figure 2. An example for 'taking no notice for the negative/positive signs while manipulating the algebraic expressions.

It can be seen by the answer of the 6th question that the participant student transformed 'y' and '10' from one side to another without changing their signs.

Figure 3 is the photo of a participant's answer sheet for 15th question. It is another example for 'taking no notice for the negative/positive signs while manipulating the algebraic expressions'.

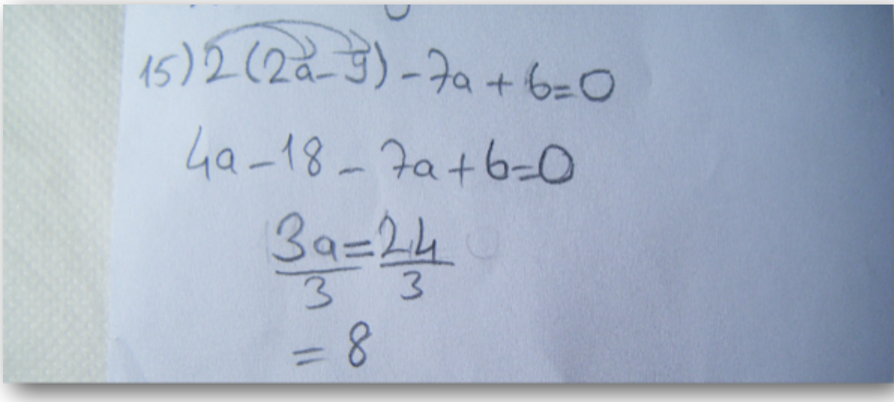

Figure 3. Another example for 'taking no notice for the negative/positive signs while manipulating the algebraic expressions.

It can be seen in Figure 3 that, the participant did not take the negative signs of '18' and '7a' into consideration while s/he was solving the 15th question. Therefore, he/she made mistake in subtracting '7a' from '4a'. The solution is also an example for 'taking no notice for the negative/positive signs while manipulating the algebraic expressions'.

In Figure 4, the misconception named as 'ignoring to use the rules for brackets in an algebraic expression' represented by a participant student's answer to the 12th question.

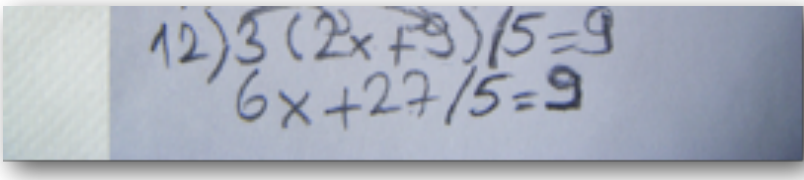

Figure 4. An example for 'ignoring to use the rules for brackets in an algebraic expression'

In Figure 4, a participant's answer to the 12th question is represented. It can be seen that, the participant ignored to divide 2x by 5. The ignorance is an example for the misconception of 'the ignorance of using the rules of brackets in an algebraic expression'.

Interview Results of the Participants

After the implementation of the question sheets, interviews were conducted with 2 participants. Participant 1 had medium-achievement in mathematics whereas Participant 2 had high-achievement in the same field. Participant 1 gave wrong answer to the first question (1st). The question was as follows:

2a + 8 = 10, find the value for 'a'?

Participant answered the question as '0'. The reason for the answer was asked to the student. He said that, "the answer is zero because 'a' is an ineffective element in that expression. Since the ineffective element of addition operation is '0', the answer is zero". Then the meaning of 2a + 8 = 10 was asked to the same participant. He said that "the addition of 8 and another number gives 10" and continued with saying that "8 is an ordinary number but 2a is different. It seems that a number is added to 2".

It can be understood from the interview that, the participant treats to the operation symbols as they are not exist in an expression. Besides, he ignores the presence of the letter and explains this situation as 'a is an ineffective element in that expression'.

The researcher also asked the following question to Participant 1: "what does x/3 + 2x/3 mean in the 5th question?". He said that 'there are two numbers added to each other and they are two digit numbers'.

In the interview it was asked to the same participant whether 2m in the 13th question (3(1 - 2m) = 4m – 5) could be subtracted from 1. He said 'yes' and added that "the answer becomes '-1'. However there is 4m in the other side of the equation. Then we should turn the negative sign between 1 and 2m to become positive sign. Then the result becomes 4m. The result of 4m - 5 is also -1". It is understood by the interview that, Participant 1 'ignored the letters in the algebraic expression'.

During the interview with Participant 2, the researcher asked the following question: "why could not you answer question 16 which is asking the value of k in 1 - 8k = 0.2 algebraic expression".

She (Participant 2) said that, there was a decimal number in the question, so she could not solve the question. According to her answer it can be understood that, based on her lack of knowledge about the arithmetical operations between decimal numbers, she was unable to make operations between the variables.

The 14th question was "find the number of unknowns in x + y + z = 30 algebraic expression". Participant 2 found the answer as "3". She explained her answer by saying 'each of the letters is used instead of a different number because there are three different letters. If all the letters were used instead of the same number, only one kind of letter would have been used'. The explanation shows that, the student has understood the reason for using different labels for each identical variable.

Discussion

The results showed that, in the study the mostly observed misconceptions about the participant students' algebraic equation operations were (a) inability of making operations between the variables based on the lack of knowledge about arithmetical operations, (b) believing that the letters are used only for the natural numbers, and (c) ignoring the letters included in the algebraic expressions.

The study also showed that not taking the letters as variables, ignoring to use the rules for brackets in an algebraic expression, treating to the operation symbols as they are not exist in an expression, and taking no notice for the negative and positive signs while manipulating the algebraic expressions are the other misconceptions which are not common but observed in the participant students' solutions for the algebraic expressions.

Inability of making operations between the variables based on the lack of knowledge about arithmetical operations was the category of the misconception which was mostly observed in the algebraic operations of the students. This result is similar with the one showed by the study of Baroudi (2006); Mulungye (2010); Norton and Irvin (2007); Stacey and Chick (2004); Stacey and Macgregor (1999), and Wu (2001). In his study Baroudi (2006) told that, the reason for such contradictions in the algebra operations was the inadequate computation variety of students' past practices.

In the study it was also seen that ignoring to use the rules for brackets in an algebraic expression was a misconception observed in the algebraic equation operations of the students. This result is similar with the results of the study constructed by Vendlinski, Howard, Hemberg, Vinyard, Martel, Kyriacou, Casper, Chai, Phelan, and Baker (2008); Akkaya and Durmuş (2006); and Booth, McGinn, Barbieri, and Young (2017). Ignoring brackets may result with incorrect answers because of the wrong order of the operations or lack of the operations done for the algebraic equation solutions.

Booth et al. (2017) observed that students ignored the presence of variables in algebraic equations like three of the participants in the current study did. Besides lots of the studies showed that students had variety of misconceptions on the role of letters in algebra like they had in the current study (Akkaya & Durmuş, 2006; Asquith et al., 2007; MacGregor & Stacey, 1997). The results of the study conducted by Şahin and Soylu (2011) that shows the belief of the students as letters has only digit values is another misconception that is also observed in the current study.

In their study about the students' dealings with the algebraic equations, most of the researchers such as Akkaya and Durmuş (2006), Asquith, Stephens, Knuth, and Alibali (2007); Clement (1982); MacGregor and Stacey (1997); McNeil, Weinberg, Hattikudur, Stephens, Asquith, Knuth, and Alibali (2010); and Usiskin (1988) observed misconception of comprehending the letters as objects. This result was slightly observed in the current study. However; conducting the study with a larger sample size may show similar results with the other ones.

According to the results of the study which was conducted by Cangelosi et al. (2013), negative sign difficulties may last during the university years, too. Thus, observing that two of the participants in the study took no notice for negative/positive signs while manipulating the algebraic expressions is also a detail that should be underlined as an important result.

In the results of the studies which were conducted by Booth and Koedinger (2008); Behr, Erlwanger, and Nichols (1980); Falkner, Levi, and Carpenter (1999); Kiearan (1992); Gonzales, Ambrose, and Martinez (2004), and Cheng-Yao, Yi-Yi, and Yu-Chun (2014), the misconceptions based on the misunderstandings about the equal sign were observed. In the current study, the categories of treating to the operation symbols as they are not exist in the expression and taking no notice for the negative and positive signs while manipulating the algebraic expressions embrace the misconception category based on the equal sign. Besides, the results are similar with the observations made by Matzin and Shahrill (2015) in their study about students' formulating and manipulating of the algebraic equations. Therefore; it can be said that the misconceptions about the wrong implementation of the equal sign, which was shown by lots of studies in the literature, was also observed in the study.

Conclusion and Implications

It can be said that the results of the study mostly supported the studies conducted in the same area. The reason of the non-similar results with some of the studies can be because of the short time interval and small sample size. Furthermore; the answer of the research question found answer in the study. Thus; it can be said that the misconceptions of the participant students about their algebraic equation practices were based on the misconceptions that they already have because of their arithmetical backgrounds, negative/positive sign contradictions, the equal sign misunderstandings, and the letter-variable controversy. On the other hand; it was observed by the interviews constructed in the study that, although some misconceptions were observed in the written formats, participant students were aware of the reasons of some of their misconceptions.

According to the results of the study some recommendations for further studies can also be given. First of all, this study was administered to the students only in the 7th grade. In the future, the research can be developed and implement to all levels of students. So, the misconceptions of any grades can be understood. Besides like NCC (1992) did, Turkish Ministry of National Education can design and implement similar but much more comprehensive study than the current one to observe and identify the common misconceptions that students and also teachers have in their algebraic thinking.

For a hundred years; it was an obligation for the teachers to be a guide to the society, to plan teaching, and to prepare materials for their lectures. In order to clear off the studies saying that, only a few teachers determined the understanding that many elementary school students hold misconceptions about the meaning of the equal sign; becoming aware of the students' learning process and taking care of the related research seem to be an obligation for education system (Stephens, 2006). Therefore; it may be beneficial to determine and solve the students' misconceptions, source of misconceptions and finding ways to fix the misconceptions at any field of mathematics.

References

Akkaya, R., & Durmuş, S. (2006). İlköğretim 6-8.sınıf öğrencilerinin cebir öğrenme alanındaki kavram yanılgıları, *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 31, 1-12.

Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle school mathematics teachers' knowledge of students' under- standing of core algebraic concepts: *Equal sign and variable. Mathematical Thinking and Learning: An International Journal*, 9(3), 249–272.

Baroudi (2006). Easing students' transition to algebra, *Australian Mathematics Teacher*, 62 (2), 28-33.

Behr, M., Erlwanger, S., & Nichols, E. (1980). How children view the equal sign, *Mathematics Teaching*, 92, 13-15.

Booth, J.L., & Koedinger, K.R. (2008). Key misconceptions in algebraic problem solving. In B.C. Love, K. McRae, & V. M. Sloutsky (Eds.), *Proceedings of the 30th Annual Cognitive Science Society* (pp. 571-576). Austin, TX: Cognitive Science Society.

Booth, J., Mcginn, K., Barbieri, C. A., & Young, L. K. (2017). Misconceptions and learning algebra. Retrieved May 2019 from https://www.researchgate.net/publication/309344387.

Cangelosi, R., Madrid, S., Cooper, S., Olson, J., & Hartter, B. (2013). The negative sign and exponential expressions: Unveiling students' persistent errors and misconceptions. *The Journal of Mathematical Behavior*, 32(1), 69–82.

Chi, M.T.H. (1978). Knowledge structures and memory development. In R.S. Siegler (Ed.), *Children's thinking: What develops?* (pp. 73-96). Hillsdale, NJ: Erlbaum.

Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education*, 13(1), 16–30.

Cheng-Yao, L., Yi-Yin, K., & Yu-Chun, K. (2014). Changes in pre-service teachers' algebraic misconceptions by using computer-assisted instruction. *International Journal for Technology in Mathematics Education*, 21(3), 21–30.

Christou, K.P., Vosniadou, S. & Vamvakoussi, X. (2007). Students' interpretations of literal symbols in algebra. In S., Vosniadou, A. Baltas & X. Vamvakoussi, (Eds.). *Re- Framing the Conceptual Change Approach in Learning and Instruction. Advances in Learning and Instruction Series,* Elsevier Press.

Davidenko, S. (1997). Building the concept of function from students' everyday activities. *The Mathematics Teacher,* 90 (2), 144-149.

English, L. Warren, E. (1998). Introducing the variable through pattern exploration, *The Mathematics Teacher,* 91 (2), 166-170.

Erbaş, A. K, & Ersoy,Y. (2003). Kassel projesi cebir testinde bir grup Türk öğrencisinin başarısı ve öğrenme güçlükleri. *İköğretim Online Dergisi*, 4 (1),18-39.

Falkner, K. P., Levi, L., & Carpenter, T. P. (1999). Children's understanding of equality: a foundation for algebra, *Teaching Children Mathematics*, 6, 232-236.

Foster, D. (2007). Making meaning in algebra examining students' understanding and misconceptions, *Assessing Mathematical Proficiency*, MSRI Publications, 53, 163-176.

Fraenkel, J. R., & Wallen, N. E. (2003). *How to design and evaluate research in education*. (6th ed.). McGraw-Hill, Inc.

Gardella, F. J. (2009). Introducing difficult mathematics topics in the elementary classroom: A teacher's guide to initial lessons. New York: Routledge, Taylor and Francis.

Gonzalez, M. M., Ambrose, R., & Martinez, E. C. (2004). In the transition from arithmetic to algebra: misconceptions of the equal sign, *Proceedings of the 28th International Group for the Psychology of Mathematics Education,* England, 1-329.

Jupri, A. & Drijvers, P. H. M. (2016). Student difficulties in mathematizing word problems in algebra. *EURASIA Journal of Mathematics, Science and Technology Education,* 12(9), 2481-2502.

Kiearan, C. (1992). The learning and teaching of school algebra. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan.

Knuth, E.J., Alibali, M.W., McNeil, N.M., Weinberg, A., & Madison, A.C.S. (2005). Middle school students' understanding of core algebraic concept: Equivalence & variable. *Zentralblatt für Didaktik der Mathematik*, 37(1), 68-76.

Koedinger, K.R., Anderson, J.R., Hadley, W.H., & Mark, M.A. (1997). Intelligent tutoring goes to school in the big city. *International Journal of Artificial Intelligence in Education, 8*, 30-43.

Küchemann, D. (1978). Children's understanding of numerical variables. *Mathematics in Scholl*, 7(4), 23-26.

Macgregor, M., & Stacey; K. (1997). Ideas about symbolism that students bring to algebra. *The Mathematics Teacher*, 90(2), 110-113.

Matzin, E. S., & Shahrill, M. (2015). *A preliminary study of year 7 students' performance on algebraic concepts.* Paper presented at The 7th ICMI-East Asia Conference on Mathematics Education, Cebu City, Philippines.

McNeil, N. M., Weinberg, A., Hattikudur, S., Stephens, A. C., Asquith, P., Knuth, E. J., & Alibali, M. W. (2010). A is for apple: Mnemonic symbols hinder the interpretation of algebraic expressions. *Journal of Educational Psychology,* 102(3), 625– 634.

Ministry of National Education (MoNE) (2005). *Öğretim Programlarının Değerlendirme Raporu*. Ankara: Milli Eğitim Basımevi.

Ministry of National Education (MoNE) (2009). *Ilköğretim Okulları Ders Programları: Matematik Programı 6-8.* Ankara: Milli Eğitim Basımevi.

Mulungye, M. M. (2010). Sources of students' errors and misconceptions in algebra and influence of classroom practice remediation in secondary schools Machakos Sub-County, Kenya. Unpublished Master's Thesis. Kenyatta University, Kenya.

National Curriculum Council (NCC) (1992). *The British National Curriculum in Mathematics.* London: HMSU Publications.

Norton, S. & Irvin, J. (2007).A concrete approach to teaching symbolic algebra. Retrieved March 24, 2010 from www.merga.net.au/documents/RP502007.pdf.

Philipp, R. (1992). The many uses of algebraic variable. *The Mathematics Teacher*, 85 (7), 557-561.

Romberg, T. A., Carpenter, T. P., & Kwako, J. (2005). Standards-based reform and teaching for understanding. In T. A. Romberg, T. P. Carpenter & F. Dremock (Eds.), *Understanding mathematics and science matters*, 3-26, New York: Routledge.

Stacey, K. & Chick, H. (2004).Solving the problem with algebra. In K. Stacey, H. Chick, & M. Kendal (Eds.), The Future of Teaching and Learning of Algebra. The 12th ICMI Study (pp. 1-20). Boston: Kluwer.

Stacey, K., & MacGregor, M. (1999).Taking the algebraic thinking out of algebra. *Mathematics Education Research Journal*, 1, 24-38.

Stephens, A. (2006). Equivalence and relational thinking: preservice elementary teachers' awareness of opportunities and misconceptions, *Journal of Mathematics Teacher Education*, 9(3), 249-278.

Şahin, Ö. & Soylu Y. (2011). Mistakes and misconceptions of elementary school students about the concept of variable, *Social and Behavioral Sciences*, 15(2011), 3322-3327.

Tenenbaum, G., Tehan, G., Stewart, G., & Christensen, S. (1999). Recalling a floor routine: The effects of skill and age on memory for order. *Applied Cognitive Psychology, 13*, 101-123.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. F. Coxford, & A. P. Schulte (Eds.), The ideas of algebra, K-12 (pp. 8–19). Reston, VA: National Council of Teachers of Mathematics.

Vendlinski, T. P., Howard, K. E., Hemberg, B. C., Vinyard, L., Martel, A., Kyriacou, E., Casper, J., Chai, Y., Phelan, J. C., Baker, E. L. (2008). Using data and big ideas: teaching distribution as an instance of repeated addition. *National Center for Research on Evaluation, Standards, and Student Testing*, University of California: Los Angeles.

Wagner, S. (1983). What are these called variables? *Mathematics Teacher*, 76, 474-478.

Wu, H. (2001). How to prepare students for algebra. *American Educator*, 25, 10-17.