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Procedia Social and Behavioral Sciences 15 (2011) 2554–2558

Procedia
Social and Behavioral Sciences

WCES-2011

Pre-service mathematics teachers' multiple representation competencies about determinant concept

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Abstract

In this study, pre-service secondary school mathematics teachers' pedagogical content knowledge on determinant subject with respect to multiple representation components was investigated. The study was carried out with 25 pre-service mathematics teachers including 10 females and 15 males. The study was descriptive and the data were gathered from the written responses to test including open ended question which was subjected to pre-service mathematics teachers and from the semi-structure interviews. The results of this study revealed that pre-service teachers experienced difficulty in taking determinants and that they used the numeric and algebraic representations which are the kinds of multiple representations, instead of geometric representations. In addition, it was found that they could not express the definition and the meaning of the determinant completely and they felt that geometric representations only helps them intuitively understand the meaning of the determinant concept.

Keywords: Pedagogical content knowledge, multiple representations, pre-service mathematics teachers, teaching linear algebra

1. Introduction

Teachers' role on breaking negative and ossified perception in learners' mind about mathematics is absolutely very important. Considering this situation from the teacher dimension, mathematics teachers' knowledge and competencies keeps an important place. There are special content competencies determined for secondary school mathematics teachers (MEB, 2009). As it is mentioned in Shulman's (1987) second report, knowledge that teachers need to possess was grouped under seven dimensions. These dimensions are;

- Content knowledge
- Pedagogical content knowledge
- Curriculum knowledge including teaching materials and programs
- Pedagogical knowledge including classroom management and organization
- Knowledge of students and their properties

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doi:10.1016/j.sbspro.2011.04.144

- Knowledge of educational environment and its availabilities.
- Knowledge of purposes, objectives, values and their philosophical and historical bases related to education.

Shulman (1986) first introduced the category of pedagogical content knowledge that was separated from the content knowledge and pedagogical knowledge that were known to be teacher knowledge before. Pedagogical content knowledge separates the expert of the content from the educator (Shulman, 1987, p.8). According to Shulman (1986), pedagogical content knowledge includes the representation types and teaching forms for others to better understand the subject taught. From this perspective, PCK includes the most beneficial representations, strongest analogies, illustrations and examples. The representation concept mentioned above is a psychological concept that explains and gives meaning to how students express their ways and forms of thinking in their mind in mathematics education area. Representation is a kind of process of how individuals perceive the external world and how they give meaning to events which are out of them with respect to their own point of views (Radford, 2001, cited in Durmuş & Yaman, 2002).

Cognitive representation that individuals construct about a specific subject can be, even will be, different. The reason is that each individual has different cultural background about the situation faced and that there occurs differences for individuals to relate the concept about such situation with their cultural background. A cognitive representation is not only individual's visual or physical end product. Or it is not physical event or concept (Radford, 2001, cited in Durmuş & Yaman, 2002).

Multiple representations became one of the important subjects to study and the attention to this subject increased in recent years. The main reason for this situation is that, apart from the other NCTM standards between 1989 and 2000, multiple representation approach as a new operational standard was developed. Especially in NCTM 2000 standards, multiple representation approach was stated and emphasized (NCTM, 2000).

> "Students should be taught the value of representing mathematics verbally in words, numerically in tables, visually in graphs, and algebraically in symbols and should learn how these various representational forms of mathematics are connected. Multiple representations also help reach the goal of algebra for all. A teaching strategy that connects the various forms of multiple representations to describe mathematics is an effective strategy for reaching out to students with different learning styles." (Choike. J. R. 2000, p.557).

Literature related to multiple representation indicated that multiple representation helps students to fully comprehend the operations while students are not aware of the meaning of the operations they did, the results they found at the end of the operations, and while they believe that the operations done and results found are meaningless and they try to learn the subject by memorization (Oaks, 1990). In this study, using multiple representations for intuiting the meaning of determinant concept and of its values to students, which are not shown as student difficulty in linear algebra course, was taught to be important to study. Although majority of the studies related to student difficulties in linear algebra were about students' operations and their mistakes in linear algebra course (Harel, 1989, Wamg, 1989, Carlson, 1993 & Dorier et al., 1994), meaning behind their operations is also important to consider.

In this study, pre-service mathematics teachers' pedagogical content knowledge about determinant subject was investigated in the context of multiple representation components. From this perspective, first of all, it was taught to determine which multiple representation type's pre-service mathematics teachers use related to the determinant concept, and their views about what the meaning of determinant value is. During the study, however, some situations necessitate how the multiple representations intuitively comprehend the meaning of the determinant and its values. Therefore, the aim of this study was shaped with the three sub research questions. These are;

- Which of the multiple representations do pre-service mathematics teachers use about the determinant concept?
- What is the meaning of determinant concept for pre-service mathematics teachers?
- How does intuitively the multiple representations in understanding determinant concept affect the pre-service mathematics teachers?

2. Method

This study is a case study which is an in-depth investigation of the situations of what determinant concept means for pre-service mathematics teachers, which types of multiple representations they use and how multiple representations intuitively affect pre-service mathematics teachers in understanding the determinant concept. The sample of this study is composed of 25 pre-service teachers (10 female, 15 male). The data for this study were gathered from an achievement test including 4 open-ended questions related to determinants of 2x2 type matrices, participants' written responses to 2 questions about the definition and meaning of determinant. After analyzing the data gathered from these data collection tools, there is a need occurred to intuit the meaning of the determinant with multiple representations to pre-service mathematics teachers. Then, semi-structured interviews were made with seven pre-service mathematics teachers. Therefore, the data were gathered from written responses, an achievement test and interviews. The data from such data collection tools were analyzed descriptively.

It was aimed to present pre-service teachers' views about definition and meaning of the determinant with the written responses, to investigate the solution of the determinant value and the usage of multiple representations with the achievement test, and to investigate the role of interviews in intuiting the conceptual structure of geometric representation which is one type of multiple representations about the determinant concept. Participants were expected to write all solution methods in finding the determinants in achievement test. Multiple representations in determinant subject was classifies as follows. Numeric (N), Algebraic (A) and Geometric (G).

3. Findings and Interpretation

The findings of this study revealed that pre-service mathematics teachers did not experience difficulty in finding the values of the determinant. They used numeric and algebraic representations, but, they did not use the geometric representation. In addition, they could not comprehensively show the definition and meaning of the determinant, and geometric representation played an intuitive role in understanding the determinant concept.

Pre-service teachers were expected to answer 4 open ended questions with all possible solution methods they knew. As it was expected, they answered correctly to all questions. They used numeric (N) and algebraic (A) representations for these 4 questions, but, never used geometric representation (G) for any of these questions. Especially for the questions (2 questions) including determinants with few numbers of components, they all used the numeric representation in finding the value of the determinants. It was found that pre-service teachers solved these 2 questions with transition from numeric to numeric representation. In numeric solutions, the solution methods for finding the value of determinant consisted usage of direct application (NT), echelon form (NE) and cofactor (NK) which are the rules of finding determinant value. 15 pre-service teachers used only NT. 6 of them used both NT and NE, and 4 of them used both NT and NK. In questions with large number of components, transition from numeric to algebraic representations was used. In such questions, 6 of them used only NT, 9 of them used only algebraic (A) representation. 4 of them used both NT and NE. In addition 6 of them used both NT and C. In these 4 questions, geometrical representation was never used.

In the interview with pre-service mathematics teachers, for the definition of the determinant, 17 of them used the taking determinant rule for 2x2 type matrices. Moreover, 4 pre-service mathematics teachers defined the determinant with the exact numerical value of the matrices, and 2 of them defined it as the function that is defined from matrices to real number. In addition, 2 pre-service mathematics teachers could not answer the questions. To responses to question related to the meaning of the determinant, 7 pre-service mathematics teachers stated that it is the solution of the equation system, 3 of them said that it is the value of matrix, 2 of them said that it is normal – distance in matrices and a pre-service teacher stated that is the determination of linear dependence and finding the solution set. In addition, 12 of them did not give answer to the question. The pre-service mathematics teachers' answers were about why they use the determinant, instead of what it means.

With the possibility of intuiting the meaning of the determinant value with geometrical representation, one of the multiple representation types, at least in 2nd and 3rd dimensions, interviews were carried out with the participants. That means, pre-service teachers were intuitively tried to be understood the meaning of the determinant value with the interviews conducted to them. Geometry used in the interview processes was seen to be helpful for pre-service mathematics teachers to understand the meaning of the determinant concept and determinant value. One of the interviews was stated as follows.

Note: Researcher used more than one matrix examples during the interview with pre-service mathematics teachers to study similar processes. However, only one example for each process was stated below.

Researcher(R): If $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, then det(A)=?

Interviewee (I): det(A)=2

R: Can you draw the row vectors of the matrix in the coordinate system?

I: (Drawn)

R: What kind of a shape that vectors constructed?

I: Probably a rectangle... yes rectangle.

R: Can you find the area of the shape appeared?

I: 2.

R: Please be careful about the value and area of the determinant.

I: Same as the area. Namely, does the value of determinant represent the area? But, is it same for each example? (A few examples were solved). Yes it is true. Then, for 3x3 matrices, it represents the volume, because there are 3 vectors.

R: Let's do it for another matrix. For example, if $B = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$, det (B)=?

I: 0

R: Can you find the area of the shape built by row vectors?

I: The area cannot be constructed. The length appeared. They are all on the same line. If the vectors constructs an area, then, the determinant is different than 0. It is already 0.

R: If $C = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$, then det(C)=?

I: -2

R: Can you do the same for this?

I: It is the same as the above example. However, the area cannot be negative.

R: So, was our statement wrong for matrix A?

I: The length could not be negative, then, the absolute value was used. It is also valid for this situation. Absolute value of determinant value is valid.

R: What is the difference between the matrices A and C?

I: Rows were displaced.

R: What did this change?

I: There is already such property in matrices. The sings change.

R: What is the meaning of this?

I: I don't know. (S/he thinks)

R: Let's solve more examples. Follow the curve from one end point of the row vector to the other end point. Be careful about the turning directions. Can you do the same thing for matrices A and C?

I: For matrix A, from (2, 0) to (0, 1). For the matrix C, it is reverse. If we take the end points, it is related to the clockwise and counter-clockwise directions. Actually, the starting points are also the same.

4. Conclusion and Recommendations

The findings of this study indicated that not all of the pre-service mathematics teachers were aware of the related multiple representations. Multiple representations would be helpful for pre-service teachers in having in-depth knowledge about concepts and what they did in operations, at least, in being aware of the meaning of what they did in studying determinants and matrices. Especially, the results figured out that knowing the multiple representations is helpful for them in understanding and giving meaning to concepts in mathematics. From this perspective, it is obvious that using multiple representations, one of the important components of PCK, is a necessity during the teaching processes.

This study can be broadened by making in-depth interviews about properties of matrix determinants and about the determinants with matrices of different dimensions. In addition, psychological reflections of this pedagogy about appropriate epistemology of determinant concept on both students and pre-service teachers can also be investigated.

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