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Semiotic representations skills of prospective elementary teachers related to mathematical concepts

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Abstract

The aim of the study is to introduce Duval's Theory of Registers of Semiotic Representations, and with the framework of this theory to analyze how students perceive and apply different usages of the representations for the same concept and transformations between these representations. The study was carried out with 28 students from Artvin Çoruh University the Faculty of Education Classroom Teacher Education program in 2007-2008 educational year. For the study, activities about identities, equations and functions were prepared and applied. Students worked as groups of two. As soon as they completed the activity all groups were interviewed in unstructured way. Students particularly can be said to have advanced transformation skills in algebraic register of representation. However, the students were not observed as good at skills of passing from verbal and graphical representations to algebraic one, and from tables to verbal representation.

Keywords: Theory of registers of semiotic representations; coordination of registers of representation; identities; equations; functions.

1. Introduction

Since mathematical concepts and relations are abstract, they can not be seen and felt in daily life (Duval, 1993, 2000). For this reason,the skill of "*expressing mathematical knowledge and thoughts by using representations such as; concrete models, illustrations, graphics, and tables etc.*" (MEB, 2005) becomes a vital skill to learn mathematics. So, it is important for students to recognize and use different representations of a concept and to shift from one presentation to another. Therefore, using different representations and transformation between these representations should be encouraged so that conceptual learning can be achieved. This can be managed by using different representations for the same concept at the same time and context (Winslow, 2003). Based upon this assumption researchers put forward certain theories related to "representation" concept. One of these theories is the Theory of Registers of Semiotic Representations.

1.1. Theory of registers of semiotic representations

Founded by Raymond Duval, the theory originated from the idea that mathematic concepts or relations cannot be comprehended directly and cannot be perceived in real life, since they are abstract (Duval, 1993, 2000). While an acid studied by a chemist and a creature studied by a biologist do really exist, an equation, straight line *etc*. do not exist in real life. For example; who can perceive the parabola; x^2-$5x$-6=0 by real life experience? The perception of mathematical concepts or relations occurs in

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the mind and operations on them are only possible by using certain signs and symbols, which renders representations inevitable for learning mathematics.

Representations are formed by assembly of certain signs and symbols obeying the well-set rules. In this approach; Algebra, daily language, graphics, tables and computer languages (Basic, Pascal, *etc*) all referred as Semiotic Register of Representation. One another point Duval concerned was the transformations within or between semiotic representations, known respectively as *treatment* and *conversion.* They can be explained as follows:

A treatment is a transformation (i.e. passing from one representation to another) within the same register. For example; paraphrasing or replacing with synonyms in daily language or adding the same number to the both sides of an equation in algebraic register *etc.* So, converting $2x-1=4$ representation to $2x=5$ or expressing $A\cup(B\cap C)$ as $(A\cup B)\cap(A\cup C)$ can be given as examples.

As for conversion it consists in transforming a representation of one register into another representation in another register with or without data loss.Translation from one language into another, expressing the problem "*There are 20 feet in a farm hosting 7 sheep and chickens. How many sheep and chickens are there in this farm?*" as equation system like $S + C=7$ and $4S + 2C =20$, drawing lines defined by these equations or tabulating these data can be given as examples for conversion.

Gagatsis & Shiakalli (2004) determined that poor representation transformation skills of students adversely affect their mathematical learning and problem solving performances and such students have weak skills of utilizing mathematical ideas and relations learned. It can be said that students' acquisition of the skills of carrying mathematical concepts or relations from one representation to another affects their learning and their problem solving strategies (Lesh ,Post & Behr, 1987). To develop these student skills, first it must be detected that to which extent these skills exist. Besides students' expression of an idea or concept *etc*. formed in their minds via representations such as tables, equations graphs helps us to get hints about some important concepts such as; how they acquire a certain piece of knowledge and the way they think (Radford, 2001). Therefore the aim of this study is to analyze students' application of these representations and how students perceive transformations (treatment or conversion) of these representations. Concerning the things above the answer for the question "*What is the prospective elementary teachers' using level of representations for different mathematical concepts and what are their level of skills of conducting these conversions?*" was pursued with the present study.

2. Method

The study was conducted in Fundamentals of Mathematics II course in spring term of 2007-2008 educational year. The sample of the study was 28 freshmen from Artvin Çoruh University Faculty of Education Elementary Teacher Education Department. 4 different activities about identities, 1st degree equation systems and functions were prepared to comply with the necessities of the study and applied. The activities prepared as to give the sample opportunities to use different representations for the same concept and make transformations between these representations. Then, the skills that the sample are to perform were determined. The participants in the sample were worked in pairs and they were given 90 minutes for each activity. Following each activity, semi-structured interviews were conducted and recorded with all groups and the groups' level of exhibiting pre-set skills were tried to be determined evaluating activity drafts of the groups and the interviews. Groups are coded by G1, G2, …….G14.

3. Data and interpretation

In this part after a brief introduction of the activities, students' level of exhibiting pre-set behaviors will be tabulated and interpreted.

1st activity had four questions. The first two questions were aiming to have students realize the geometrical meaning of identities and to construct student knowledge about conceptual meaning of identities by making students apply treatment within geometric or algebraic representations and conversion between these representations. And the 3rd and 4th questions were for stating the students' application level of the knowledge they constructed with the help of the instructions given in the first two questions.

As the table indicates while all the groups succeed in reaching the expression which is identical to the square of sum of two terms utilizing geometrical representations, only four groups managed the same for the square of difference of two terms. Another highlight of the table is that none of the groups could demonstrate the skill of finding the square of difference of two terms utilizing treatment within geometry.

Similarly, all groups primarily preferred writing an identical expression to square of sum of three terms in an algebraic point of view. 7 groups preferred treating the expression into the square of sum of two terms by replacing $a+b$ with x in $(a+b+c)$ expression and then they applied their usual solution steps. G5, G10 and G13 preferred to write $(a+b+c)$ twice and to multiply directly as algebraic solution. G7 and G4 did not submit any algebraic solutions. Since the students were asked to solve the problem with different means, the groups utilized geometric representation as the alternative way.

As the table shows, six groups achieved to find the square of sum of three terms using geometric representations. They headed to the solution by drawing a square whose sides are $(a+b+c)$ units, and adding the areas of separated rectangular areas. G4, G10, G11, G12 and G13 associated $(a+b+c)^2$ expression with area of the square with $a+b+c$ unit side-length but while they were solving problem they simply used algebraic methods (to write $(a+b+c)$ twice and to multiply).

Table 1. The state of encountering expected skills on students about identity topic

Skill	Activity 1	Skill encountered at	Skill partly encountered at	Skill not encountered at
Finding the square of sum of two terms utilizing geometric representations and conversion from geometry to algebra.	Question 1	G1,G2,G3,G4,G5, G6,G7,G8,G9	G10,G11,G12, G13,G14	
Finding the square of difference of two terms utilizing geometric representations and conversion from geometry to algebra.	Question 2	G1,G2, G8, G13		G3,G4,G5,G6,G7,G9, G10,G11,G12, G14
Finding the square of difference of two terms utilizing geometric representations and treatment within geometry	Question 2			All groups
Finding the square of sum of three terms utilizing algebraic representations and treatment within algebra.	Question 3	G2,G3,G5,G6,G8,G9, G10,G11,G12,G13,G14		G7,G4
Finding the square of sum of three terms utilizing geometric representations and conversion from geometry to algebra.	Question 3	G1,G2,G3,G5,G8,G14	G4,G10,G11, G12,G13	G6, G7,G9
Finding the cube of sum of two terms utilizing algebraic representations and treatment within algebra.	Question 4	G1,G2,G3,G5,G6, G7,G8,G9,G10,G11, G12,G13,G14		G4
Finding the cube of sum of two terms utilizing geometric representations and conversion from geometry to algebra.	Question 4		G1,G2,G3,G5,G6 ,G7,G8,G10,G11 ,G12,G13,G14	G4,G9

The students were asked to express the cube of sum of two terms in different way in the activity related to identity. The groups primarily preferred algebraic solution, as well. While G4 came up with no solution, the rest of the groups solve the problem in algebraic way by representing the third order expression as the multiplication of a second and a first order expression. Groups, more or less, tried to reach to the solution by forming a cube with *(x+y)* units side-length and taking each edge as *x* and *y* and adding separate volumes in the cube. However none of the groups could reach a clear solution. G1, G2, G3, G5, G6, G7, G8, G10, G11, G12 and G14 managed to resemble the $(x+y)^3$ expression to the volume of a cube with x+y units edge length in a geometrical point of view. But except for G10 and G11, the groups could do nothing but multiplying in algebraic way. Whereas G10 and G11 tried to split the cube into smaller compartments and sought a geometrical solution by adding the volumes of all sub-compartments. They could not reach the right solution due to splitting the cube incorrectly though.

2nd and 3rd activities were designed for prompting students' skills of using different representations to solve a first order equation system with two unknowns with the help of a real life problem. 2nd activity employs transformations between algebraic (replacing, simplifying), table and geometrical (crossing point of the lines defined by the equations) representations but 3rd activity demands transformations between verbal expression, algebra and geometry. In the 2nd activity two algebraic expressions were directly given to students, while in the 3rd activity equations were not given directly, embedded into verbal expressions instead. Another difference between these activities is the number of equations. There are only two equations in the activity 2. However in 3rd activity the number of equations is increased to three to determine whether there is a correlation between the number of equations and transformation skills. In addition, in both activities it was aimed to determine students' level of conversion from tables and graphics into verbal interpretation. Since the activities are closely related, analyses were given on the same table (Table 2).

As mentioned above, in 3rd activity algebraic expressions were not given to students directly but they were hidden in the text. Students were expected to exhibit conversion skills from verbal expression to algebraic one. It can stemming from the table 2, that 7 groups were observed as exhibiting the skill of converting verbal expression into proper algebraic form using unknown and the rest of the groups could not perform it. The rest failed to set the suitable algebraic expression, tried to reach to the solution arithmetically by assigning some number values. They could not fully achieve that, either. From these data it can be extracted that students have poor skills of converting from verbal expressions to algebraic form.

Table 2. The state of encountering expected skills on students about first order equations topic

Skill	Activity	Skill was encountered	Skill was partly encountered	Skill was not encountered
Conversion from verbal expression into algebraic expression	3	G1,G2,G3,G5,G8,G11,G13		G4,G6,G7,G9,G10,G12,G14

Working out algebraic solution.	2	G1,G2,G3,G5,G6,G7,G8,G10,G11, G12,G13,G14		G4,G9
Treatment within algebra	3	G1,G3,G5,G8,G13		G2,G4,G6,G7,G9,G10,G11,G12,G14
Conversion from algebraic expression to verbal expression	2	G1,G2,G3,G5, G8, G10,G11,G12,G14	G6,G7,G13	G4
	3	G1,G5,G8	G9,G10,G11,G13	G2,G3,G4,G6,G7,G12,G14
Conversion from table to verbal expression	2	G1,G5,G6,G8,G10,G11,G14		G2,G3,G4,G7,G9,G12,G13
Conversion from algebraic expression to graphic	2	G1,G2,G3,G5,G6,G7,G8,G9,G10, G11,G12		G4
	3	G1,G5,G6,G9	G3,G7,G11,G13	G2,G4,G8,G10,G12,G14
Conversion from graphic to verbal expression	2	G1,G2,G3,G5,G6,G7,G8,G9,G10, G11,G12,G13		G4,G14
	3	G1,G9	G3,G6,G7,G13	G2,G4,G5,G8,G10,G11,G12,G14

It was observed that students' ratio of working out algebraic solution in activity 2 is greater than it was in activity 3. When this situation is evaluated in the context of students' skills of writing algebraic expressions based upon verbal explanations, the difference in the ratio can be attributed to students' direct exposure to the algebraic expressions. Another highlight of the table is that only half of the groups could perform the skill of reaching the solutions using the given table. 7 groups submitted no comments about the solutions utilizing the table. G4 reported the reason for that as "having difficulty in interpreting since that was the first time that they had faced with this way of solution."

The students' skill of drawing graph is higher in ratio in the 2nd activity than the 3rd. It can be explained by greater number of equations in the activity 3. As a matter of fact in the 2nd activity only two equations were given, while the relations among three equations were asked to be interpreted in the 3rd activity. Similarly it can be said that the skill of realizing that common solution of the two equations means crossing point of the lines defined by these equations (associating algebraic– graphical) decreases as the number of equations increases. According to this, in the 3rd activity even students could convert from algebraic expression to geometric one, they seemed to have difficulty to interpret the graphs. Therefore it can also be said that students' skill of converting from graphic to verbal expression decreases as the number of equations increases. This was not happen only for interpreting graphical solution, but also valid for algebraic one. Details could not be observed in the table but G6, G7, and G13 could not explain the meaning of their answer for the activity 2 even though they conduct algebraic solution. G6; after forming the table and G7 and G13; after working out graphical solution could overcome their weaknesses and managed to interpret the algebraic solution. This situation can be interpreted as students may develop an explanation to a case that they could not explain in one representation, after utilizing a different representation.

In the 4th activity, the groups were given six parabolas and 10 algebraic expressions that may be for parabolas. Then they were asked to match the parabolas and equations. As it can be understood from the table 3, the students have tendency of analyzing and choosing the suitable algebraic expression for the parabola instead of trying to write the algebraic expression belonging to the parabola. 8 groups achieved to match the parabolas by testing the algebraic expressions in terms of orientation of the arms, coordinates of the vertex, and the points crossing the axes. Besides, 4 groups managed to find the right algebraic expression directly by applying conversion from the graphics to Algebra.

G1 and G10 out of those 4 groups chose the suitable algebraic expression by assigning the vertex in $f(x)=a(x-r)2+k$ form and calculating the a coefficient by using the point on which parabola crosses y axis. G3 and G6 reached to some algebraic expressions in the form of $f(x)=ax2+bx+c$ by using the vertex and the points crosses the x and y axes and then treated the given expressions into the same form then they matched. G3 and G6 performed conversion from geometry to Algebra but G7 and G14 performed conversion from Algebra to geometry. They converted the given equations into $f(x)=ax2+bx+c$ form, then they tried to match these equations with the related parabolas. G1, as an alternative way, and G2, examining the algebraic expressions one by one, chose the suitable parabola according to vertex of the parabolas, the points crossing the axes and the status of the coefficient a.

Table 3. Group distribution of the expected student skills for functions topic

Skill	Activity	Groups exhibiting the skills
Finding the suitable algebraic expression by converting graphics to Algebra.	4	G1,G6,G3,G10
Choosing suitable algebraic expressions by testing orientation of arms, by determining vertex coordinates and the points crossing the axes.	4	G8,G13,G5,G4, G11,G12,G9,G14
Matching $f(x)=a(x-r)^2+k$ with the suitable graph after transformation to $f(x)=ax^2+bx+c$.	4	G7, G14
Choosing suitable graphic for a given algebraic expression; conversion from algebraic expression to graphic.	4	G1, G2

4. Results

Prospective elementary teachers' skills of using different representations and transforming these representations were examined in the present study. Available data supported that candidate classroom teachers' skills of using different representations and transforming these representations vary depending upon the topic and the nature of the question (Çıkla, 2004). Along with this, the present study also determined that skills of using algebraic representations are dominant over skills of graphical expression and forming table skills. In this context, it was determined that students exhibits algebraic representations skills rather than geometrical representation ones and they generally apply treatment within algebra. Also in the equation subject, it can be said that the ratio of performing algebraic solution is higher. This situation can be commented as preferring algebraic form to visual form as Eisenberg and Dreyfus reported (cited by Özgün-Koca, 2008). Additionally, it was observed that the candidate teachers had difficulties in transforming from verbal and graphical representations to algebraic ones, from table to verbal representations.

It was observed that when students were given function graphs and algebraic expressions possibly related to these graphs, they failed to state the algebraic expression related to these graphs. The students preferred to examine parabolas and algebraic expressions together instead of directly stating the expression belonging to the related parabola when they were expected to match certain parabolas with their possible correspondent algebraic expression. Similarly, in the study by Kierran (1992) students were given certain graphs and they were expected to state the symbolic form of these graphs and the students had difficulties in expressing symbolic forms (cited by Özgün-Koca, 2008).

Another extending result of the present study is that the students' transformation skills from algebraic representation to graphical one are affected by number of equations. Actually when the number of equations increased 2 to 3, student interpretations got poorer. Similarly, it can be said that the skill of realizing that the common solution of two equations is the same as the crossing point of the lines defined by these equations decreases when the number of equations increased. So it can also be said that the students' conversion skills from graphical representations to daily language decreases as the number of equations increases.

The findings showed that students can comprehend a situation which they previously could not understand in another register of semiotic representation. Therefore, it can be said that using different representations foster deep understanding and conceptual learning by reinforcing different student ideas and skills (Winslow, 2003; Ainsworth *et.al.*, 1997; Even, 1998; Duval, 1993; Durmuş and Yaman, 2002).

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