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Secondary school mathematics teachers' approaches to students' possible mistakes

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Abstract

One of the important components that shape both the pedagogical and mathematical content knowledge is the component of being aware of students' mistakes. This study tries to investigate in what degree mathematics teachers correctly built their mathematical content knowledge by using the component that shapes both their mathematical and pedagogical content knowledge. According to this aim, teachers were subjected to a test that includes three questions. The content of the test include incorrect solutions of questions related to sequences, derivative and integration and teachers were asked to explain the reasons of such mistakes based on mathematical knowledge. It was assumed that there is a direct relation between their determination of the reasons of the incorrect solutions and their mathematical content knowledge on such subject. The situation was presented descriptively. The data gathered in the study revealed that mathematics teachers experienced difficulty in determining and in explaining the reasons of the incorrect solutions in general, even, some of them have different mistakes on subject.

Keywords: Mathematics teachers, subject-matter knowledge, mathematical content knowledge, pedagogical content knowledge, error, mistake approach.

1. Introduction

Different researchers proposed necessary qualifications of a good teacher and the components that shape the teachers knowledge in various ways. As it is mentioned in the second report of MEB (2009), Shulman (1987) grouped the knowledge that teachers need to have into seven sub-dimensions. One of the most noticeable dimensions is pedagogical content knowledge.

Shulman (1986) argued that pedagogical content knowledge could be considered as the integration of pedagogical knowledge and content knowledge of a specific subject. This integration, pedagogical content knowledge, is a kind of planning and introduction in order to teach such subject.

Pedagogical content knowledge proposed by Shulman (1986) includes the knowledge of learning difficulties and what makes the learning the subject easier or difficult. In addition, this component is composed of teachers and

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students' understandings, approaches and ideas on any subject. In addition, it includes the content of conceptualization of individual differences that students have during the learning process (Magnusson, Krajcik and Borko, 1999). According to Shulman (1986), teachers should do more that only introducing and transferring the accepted facts of related subject to students. A teacher needs to have the pedagogical knowledge, knowledge of content of related subject, that of curriculum and possible mistakes, that of measurement and evaluation, that of teaching techniques and strategies. In addition, teachers need to understand student characteristics and structure of learning environment. In teaching mathematics, however, this type of knowledge is considered as the knowledge that separates mathematics educators from mathematics expert (Akkoç, Özmantar and Bingölbali, 2008).

Pedagogical content knowledge requires *the interpretation* and *transformation* of teachers' mathematical content knowledge in a way to make the students' learning easier and understand the students' common learning difficulties (Van Driel, Verloop and De Vos, 1998). Teachers' expertise basis on the teaching is the ability to transfer the content knowledge of a specific subject to students in a way that students easily understand the subject (Zeidler, 2002). Teachers' pedagogical knowledge and content knowledge related to subject need to be under the dimension of "teachers' understanding of the students" (Cochran, DeRuiter and King, 1993). In addition, teachers need to develop their pedagogical content and mathematical content knowledge in line with the understanding the students' needs (Cochran, DeRuiter and King, 1993). Especially, intuitively realizing the mistakes and why they are incorrect is important in teaching-learning process. This provides teachers with determination of students' thinking strategies (Konyalıoğlu et al, 2010).

The teaching profession is not an ordinary job, instead, it requires expertise. Therefore, teachers need to be able to understand what students think without too much effort (Moreira, 2002). To do this, they especially need to determine students' mistakes in the subject taught. Determination of possible mistakes requires a good level of content knowledge about the subject. It is a necessity for teaching to have knowledge about the mathematical concept knowledge and what are the common misconceptions (NCTM, 1989; 1991). In addition, the most important factor that affects the pedagogical content knowledge is the misconceptions that students have (Park and Oliver, 2008). Teachers who are aware of the reasons of students' misconceptions on the basis of mathematical content knowledge will be able to better organize their pedagogical knowledge and to make the teaching-learning process more effective (Konyalıoğlu, 2010). Accordingly, Türnüklü (2005) who conducted a study with pre-service mathematics teachers found that there was a relation between their mathematical content knowledge and pedagogical content knowledge. Boz (2004) investigated future's teachers' content knowledge on variables subject and their pedagogical knowledge based on such subject. The researcher proposed situations that could be considered to be an evidence for relation between content knowledge and pedagogical knowledge.

Students who can correctly solve a mathematical problem need to realize and determine the incorrect solutions of similar problems, even if the result of the question is correct; and they need to explain the reasons of the incorrect solutions and mistakes. If students could do it, they meaningfully understood the related concept (Konyalıoğlu et al., 2010). Learning is not only directly related to giving correct answer, but also determining the mistakes and incorrect solutions. One of the indicators for knowing is to know the mistakes and misconceptions, if any, in order to determine whether something is correct or not.

This study mostly considered the mistake component which was one of the components that shapes the mathematical content knowledge, therefore, the pedagogical content knowledge that mathematics teachers need to have. It was tried to present how correctly mathematics teachers construct their mathematical content knowledge by using the possible student mistakes which is one of the components that shapes both mathematics' teachers' mathematical content knowledge and pedagogical content knowledge.

2. Method

In the study, an existing situation was tried to be described. Therefore, a qualitative approach was used in the study. The sample was composed of 11 mathematics teachers from different high schools in Erzurum city. Purposive sampling method, one of the non-random sampling methods, was used. Among them, two teachers had 1-5 years, four teachers had 5-10 years, three teachers had 10-15 years and two teachers had 15-20 years teaching experiences.

Teachers were subjected to a test of three open-ended questions with incorrect solutions. To gather detailed data from teachers, teachers were asked to fill the open-ended questions with the purpose of getting their views about the incorrect solutions of the first test. There was one question in the written from for each question in the test. Therefore, there were three open-ended questions in the written form which seeks for teachers' views about the incorrect solutions. The questions asked in the test for the study were appropriate to the objectives of MEB Teaching Program and Guidebook for Mathematics Course for 9^{th} -12^{th} classes (2005). First question was parallel to the objectives related to sequence sub-learning area under learning algebra. The second question was parallel to the objectives related to derivative sub-learning area under basic mathematics learning area. The last was parallel to the objectives related to indefinite integration sub-learning area under basic mathematics learning area. In fact, first question was taken from the guidebook's examples of learning areas. It was obvious that there were mistakes and incorrect solution in all of three question asked in the test. With these questions, mathematics teachers' determination of the mistakes was evaluated in line with their mathematical content knowledge. Moreover, they were asked to solve these questions correctly. With the written form, answer to the question below for each question in the test was tried to be gathered:

- Can you explain the reason(s) of this (these) mistake(s) in line with the mathematical knowledge?

The data were analyzed descriptively.

3. Findings and Interpretation

Question 1: $1+2+2^2+2^3+\ldots=?$

Solution: $1+2+2^2+2^3+\ldots=1+2(1+2+2^2+\ldots)=x \Rightarrow\ 1+2.x=x \Rightarrow\ x=-\frac{1}{2}$

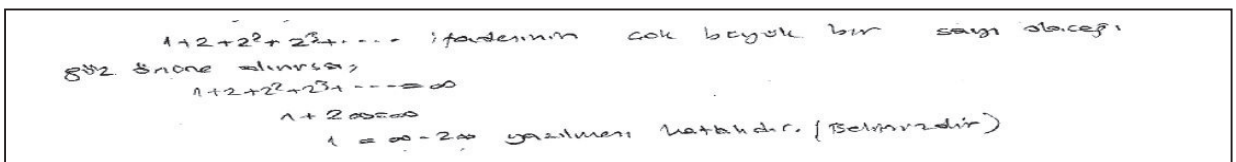
The first question in the study was related to finding the sum of a infinite geometrical sequences. The mistakes done in this question and mathematics teachers' views about the reasons for such mistakes were presented according to teachers' teaching experiences as follows.

Both of teachers with 1-5 years teaching experiences asserted that the result of this question is infinity and it is not possible to give the value of x to the parenthesis in the expression $1+2(1+2+2^2+\cdots)=x$, therefore, the mistake comes from here. They expressed the indefinite, since it is possible to subtract the infinite from infinity, if the value of x is infinite.

Similar to the responses of the teachers with 1-5 years teaching experiences, a teacher with 5-10 years teaching experience also asserted that the result is infinity, but the mistake comes from the subtraction of ∞ from ∞. Other three teachers argued that the reason of the mistake is that the sum of the sequence is divergent and they solved the question correctly.

A teacher with 10-15 years teaching experiences argued that result of -1/2 is incorrect and the correct answer is -1. Other teachers answered the question correctly.

It is very interesting that a teacher with 15-20 teaching experiences interpreted the mistake from operational dimension.



English Version: If we consider that the expression of $1+2+2^2+2^2+\ldots$ *is a very large number;* $1+2+2^2+2^2+\ldots=\infty \Rightarrow 1+2\infty=\infty \Rightarrow 1=\infty-2\infty$ *Writing like this is incorrect (Indeterminate).*

Figure 1. Solution Example for First Question

Question 2. *Find the derivative of the function* $y=f(x)=x^2$

Solution:

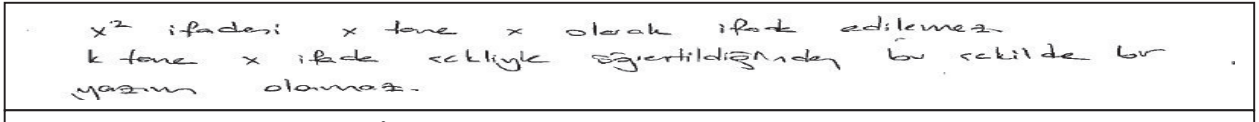
$$y = f(x) = x^2 \Rightarrow f'(x) = (x + x + x + ... + x)' \quad (x \text{ times})$$
$$\Rightarrow f'(x) = (1 + 1 + 1 + ... + 1) \quad (x \text{ times})$$

In the second question asking the derivative of the function $y = f(x) = x^2$, teachers' approaches to mistakes were as follows:

Two teachers with 1-5 years teaching experiences explained the mistake in such a way that since the sign of x could be negative, it is not possible to write x times x.

One of the teachers with 5-10 years teaching experiences stated that the in which interval the function is defined is not clear, therefore, x times x cannot be written. One other, on the other hand, said that mistake may come from inadequate knowledge of derivative and power concepts. One of the last two teachers found the mistake and tried to explain it by giving counter examples. On the other hand, the other teacher correctly explained the mistake.

Two teachers with 10-15 years teaching experiences said that x times x cannot be written, but could not explain the reason. Third teacher argued that the derivative of this function had to be 2x by explaining it with limit, but, stated the reason of the mistake by focusing only on the result.



English Version: The expression of x^2 cannot be represented as x times x. Since it is taught like k times x, it cannot be written in such way.

Figure 2. Solution Example for Second Question

One of two teachers with 15-20 years teaching experiences again explained the mistake based on the result, but could not say anything about the reason of such mistake. Other teacher explained the mistake by examples.

Question 3. $\int \tan x dx = ?$

Solution:

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx \quad \text{Here } \frac{1}{\cos x} = u \Rightarrow \tan x.\sec x\, dx = du \quad \text{and } \sin x\, dx = dv \Rightarrow -\cos x = v$$

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx = \frac{1}{\cos x}(-\cos x) - \int (-\cos x)(\tan x.secx) dx$$

$$\int \tan x dx = (-1) + \int \tan x dx \Rightarrow 0 = -1$$

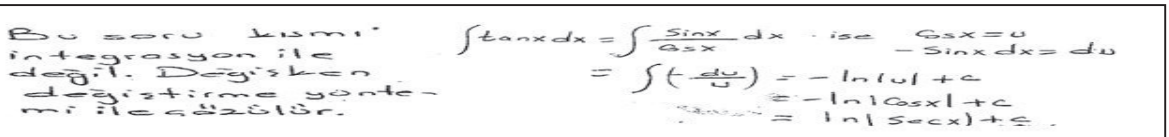
Teachers' responses to mistake on third question including indefinite integration of tan*x* were as follows:

Considering teachers' answers to this question, one of teachers with 1-5 years teaching experiences stated that s/he could not determine the mistake, while other indicated that the value interval of x is not stated, therefore, cos and sec may include indefinite values, so there may be mistakes in the simplification part.

One of teachers with 5-10 years teaching experience argued the mistake in the usage of partial integration in this question, while two others stated the necessity to apply substitution method to this question. One teacher, on the other hand, stated the reason of the mistake due to the assumption of same constants in the integration. Therefore, such teacher found the correct answer.

Two teachers with 10-15 years teaching experiences stated that they could not realize the mistake in this question. Third teacher stated the incorrect application of substitution method in this integration, and s/he again applied the substitution method which s/he taught to be correct.

One of two teachers with 15-20 years teaching experiences stated that substitution method had to be used in this integration, instead of partial integration method. Other teacher, on the other hand, argued that incorrect substitution was performed during applying partial integration.



English Version: This question can be solved with substitution method, instead of partial integration.

Figure 3. Solution Example for Third Question

4. Conclusion and Recommendations

Although the questions asked in this study were based generally on operations, performing correct operations and finding correct answers require conceptual knowledge. Being aware of possible student mistakes can provide teachers with correctly shaping both operational and conceptual knowledge. According to the findings of this study, being aware of students' mistakes, one of the PCK components takes an important place to shape their mathematical content knowledge.

It can be recommended to conduct studies that try to determine teachers' assessment and evaluation knowledge by using this study's questions including correct and incorrect solutions, or by using similar ones. In addition, teachers' content knowledge of a specific subject, being aware of mistake which is one of the PCK components, multiple representations and assessment-evaluation can be evaluated and investigated as a whole or separately by using such questions or similar ones.

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